# Department of Computer Science, University of Otago



Te Whare Wānanga o Otāgo

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## **Representing Symbolic Logic in an Artifical Neural Network**

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# Representing Symbolic Logic in an Artifical Neural Network. Technical Report I.

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**Abstract:** We report detailed experimental results for the paper 'Representing Symbolic Logic in an Artificial Neural Network, Part I: the Static Case'.

## **1 Introduction**

This technical report provides supporting information and experimental results for the paper "Representing Symbolic Logic in an Artificial Neural Network, Part I: the Static Case". The paper is concerned with finding an artificial neural network representation for supra-classical logic.

The technical report is divided into four parts. Section 2 Micro-Worlds: gives information about the experimental environment. Section 3 Boltzmann Machine: gives some features of the machine implementation and experimental results. These results support the Boltzmann machine as a viable candidate for the representation of supra-classical logic. Section 4 Multi-Layer Perceptron (MLP): looks at these experimental results with discussion about the implementation, which is not included in our paper. These results indicate that simple feed-forward networks do not have the properties required to represent the logic.

#### 2 Micro-worlds

We attempted to find a neural network representation by training and testing our candidate machines in the limited environment of logical micro-worlds. Some discussion and justification for this approach is given in our paper. The logic for these worlds was loosely based on the idea of a room, which had its temperature regulated for a minimal energy cost. A set of micro-worlds of increasing size and complexity were designed by adding atoms and preference levels. The design details of these 3, 4, 5, and 6-atom micro-worlds are given in Appendix A.

For each world a short description of the atoms and default rules are provided in the appendix, with the consequent ranked interpretation. The interpretation of the default rules is somewhat arbitrary and alternative worlds would be possible. We investigated the effect of arbitrary changes in interpretation within the 5-atom world by looking at two alternatives to the world shown in Appendix A. The changes did not affect the ability of the networks as representations. In theory such a change should not have any impact, as the networks tested were trained on the final interpretation alone.

The semantic design of these worlds deliberately involved dependency between some variables and this resulted in occasional outputs from the machines that were unexpected. The ability of the Boltzmann machine to learn and represent atomic probability and not just state distribution will be the subject of a separate paper.

It should be noted that the machines are trained using binary logic and tested using ternary logic. During training the machines are only given inputs: +1 for true and -1 for false. Model states are shown in adapted binary format, for example in the 3-atom world (Light-Fan-Heater) state "-1, 1, 1" stands for the state with "Light off, Fan on, Heater on". We will frequently use an abbreviated decimal format; in the example above, the state would be represented as decimal 3 (the two lowest order bits are on). When testing, the machines received inputs of 1, -1, or 0 for unknown or equivocal. For example the input state/query "0, 1, 0" stands for the premise "Light not observed, Fan on, Heater not observed", which has no equivalent in real binary.

Generally four alternative training sets were used for each world. Two training sets have the patterns (model states) at exponentially increasing frequency of preference and two have an arithmetic distribution. Within each pair, one set has the least preferred model states absent; the other has them present for one pattern only. The distribution of patterns in each training set is given in Appendix A, both as the actual number of patterns and the percentage of the set.

## **3 Boltzmann Machine**

#### 3.1 Implementation

As indicated in our paper, the initial implementation of the Boltzmann machine was based on Hinton's 424 Encoder. Early experimentation with this implementation involved diagnostic data sets, both mechanical and medical, with a variety of data relationships: both many-to-one (mathematical functions) and one-to-many (non-functions).

The machines performed tolerably well on even the most difficult medical data set which contained 8 diagnoses spread over 13 symptoms, with a large proportion of one-to-many relationships. The data set consisted of 120 training patterns; the machines were trained on a subset of 80 patterns.

- Tested on previously seen patterns, machines achieved a recall (correct set of diagnoses, although not always in the correct order of preference) of 88-94%.
- Tested on unseen patterns, machines had a credible accuracy of 73-78% (depending on the pattern tested).

These data sets still had a large component of classification as part of a mixed task and, at this stage, the weights of the network were not converging during training. Several methods of weight constraint were attempted including:

- Weight decay: Use of a penalty term (c = 0.001) multiplied by the total size of each weight layer matrix was partially effective in limiting the weight expansion to 1.12.
- Sparcity was implemented using a target and penalty cost as per Hinton (1985). However, this is probably only appropriate in a Restricted Boltzmann machine and was not effective in our case.
- Momentum: The standard use of momentum was most effective in limiting weight expansion and improving performance generally.

We refined the task as part of the micro-world experiments, to remove any classification component. The machines were simply required to learn the distribution of patterns (model states) in their environment. Under this task there was rapid adaption of the weights to a plateau within the first few hundred epochs, followed by a gradual linear increase in the larger inter-layer weights. We have not confirmed whether different machines converged initially to the same weight matrix for the same problem. It is likely that several different weight matrices might be possible given the nature of the optimisation problem and the stochastic nature of the Boltzmann machine.

A brief overview of the object-oriented design and the changes made to the learning algorithm of our Boltzmann machine are given in Appendix D.

#### 3.2 *Results in General*

Tuning of the Boltzmann machine was difficult. The detail of the annealing schedules for the Boltzmann machine can critically influence results. A wide variety of temperature ranges and cycling schemes were experimented with: both at high "40 down to 10" and low "5 down to 1" temperatures with long and short ranges for varying numbers of cycles at each temperature point. There was no one single correct schedule. The other tuning parameters were on average: training time 2,000 epochs, learning rate 0.3, momentum 0.7, with 20 samples per pattern. Alterations in the number of hidden nodes were investigated, for each micro-world, but the final

numbers were: 4 in the 3-atom world, 6 in the 4-atom world, 8 in the 5-atom world and 10 in the 6-atom world. The detailed results for testing of the Boltzmann machine are shown in Appendix B.

For each micro-world, testing of the neutral premise is discussed first. Five runs of 60 machines per run are given for each of the four different training sets, for each micro-world with the aim of retrieving the complete ranked interpretation. This gives a total sample size of 60,000 samples. The actual number of output samples and the percentage are shown in the tables. These results are then accumulated at the end of a row to give the average result and standard deviation for the errors over 300 machines. The sampled percentage output can be compared directly with the training distributions, on the left of each row.

Second, a selection of more specific interesting premises are shown in an attempt to demonstrate how well (or otherwise) a run of 60 machines performs the task of model selection. Adequate training of the machines is demonstrated by inclusion of some complete premises (without uncertainty). Others require a range of preference levels in the output or a single preference level. In this setting the results need to be considered qualitatively. The correct model selection within the ranked interpretation is provided, with the expected input distribution, for comparison with the actual output of the machines.

We have not however, provided the accumulated error figures for the specific premises, as these calculations were often misleading. The logic does not require exact numerical output: there were specific premises where the ranking from the logic was correctly represented by the machine but the numerical output was inexact and more importantly, there were results where the numerical output was accurate but the representation of the logic was not what was expected. These interesting premises will form the basis of a discussion of the representation of atomic distribution in a further paper.

#### 3.3 *3-Atom Micro-World*

The results of testing the Boltzmann machine against the neutral premise in the 3atom micro-world were uniformly satisfactory (Figures 5 & 6). The average error per state on the worst training set with 44 patterns was just over 2% and on the best training set with 30 patterns was less than 0.5%, with a standard deviation of  $\pm 0.11$ . When the least preferred models were present in a training set, their probability tended to be overestimated slightly by the network: 3-5% actual output versus 2.9% expected. In this circumstance, the probabilities of more preferred patterns were slightly underestimated.

The results for testing against specific premises are shown in Figure 7. The training set with 34 patterns was used for this testing: it has an arithmetic distribution and the least preferred model states are present. Looking at the completely specified premise "-1, -1, 1" it can be seen that the machine was adequately trained for the more preferred patterns (decimal state 1 - output 95%), but considering premise "1, 1, -1" not well trained for the least preferred patterns (decimal state 7 - output only 54%). The machine had difficulty adequately learning/retrieving the least preferred patterns in this micro-world, despite adjustments in the training sets and tuning parameters. A further example of this problem can be seen with premise "1, 1, 0" where the machine is 'asked' to select two of the least preferred model states. An appropriate selection would be (6) and (7) but the machine also outputs samples of (4) and (5).

However, this was not a problem when the machine was 'asked' to rank model states at different preference levels. For example, premises "-1, 0, 1", "1, 0, 0" and "0, 0, 1", in these cases the machine clearly and correctly distinguished between the probability of the most, intermediate and least probable model states. What is more, the degree of separation between preference levels was qualitatively appropriate considering the ranked interpretation.

#### 3.4 4-Atom Micro-World

The results of testing against the neutral premise in the 4-atom micro-world were also uniformly satisfactory (Figures 8 & 9). The average error per state on the worst training set with 96 patterns was under 2% and on the best training set with 88 patterns was just over  $0.5\% \pm$  one standard deviation 0.07. As for the 3-atom micro-world when the least preferred models were present in the training set, their probability tended to be overestimated slightly by the network: 2.5% actual output versus 1% expected and the probabilities of more preferred patterns were then slightly underestimated.

The results for testing against 28 specific premises are shown in Figures 10 - 13. The training set with 98 patterns was used for this testing: it has an arithmetic distribution and the least preferred model states are present. Looking at the fully specified premises; "-1, -1, -1", "-1, -1, 1", "1, -1, -1, -1" and "1, -1, -1, 1" it can be seen that the machine was adequately trained across the whole distribution (a most preferred state, decimal 0 - output 92% and a least preferred state, decimal 6 - output 85%). The machine did not have the same difficulty with learning the least preferred patterns present in the 3-atom micro-world.

The vast majority of the results for the individual specific premises indicate that the machine can select appropriate preferred model states. For example, consider premises; "0, 1, 0, 0", "0, 0, 0, 1", "-1, 0, 0, 0", "0, 0, -1, 1", "0, -1, 0, -1", "1, -1, 0, -1" and "-1, 0, -1, -1". The machine not only selects the correct model states placing them in the correct ranking, but it also maintains a qualitative degree of separation between states that might be expected from the levels in the ranked interpretation.

However, there were some exceptions to this expected behavior: for example premises "0, 0, -1, 0" and "-1, 0, -1, 0". In these circumstances the machine 'performs a tie-break' between models that should have the same state frequencies. We believe in these situations the machine demonstrates it ability to learn not just state but atomic distributions and to retrieve information related to atomic dependency within its training set. As mentioned earlier, this will be the basis of discussion in a further paper.

#### 3.5 5-Atom Micro-World

The results of testing against the neutral premise in the 5-atom micro-world were mixed (Figures 14 - 17). The average error per state was maintained at an acceptable level: on the worst training set with 158 patterns under 2% and on the best training set, with 120 patterns, just over  $1.5\% \pm 0.05$ . However, these numbers exaggerate the accuracy of the machine in this world. On all the training sets the machine grossly underestimated the probability of the most preferred model states. Moreover testing in the training sets with the least preferred patterns absent, demonstrated that the design of this micro-world had many dependent variables, the machine was 'tiebreaking', even where the premise made no observation of the environment. Initially this was felt to be unacceptable.

It was decided for completeness however, that 12 more specific premises would be tested (Figures 18 - 20). The training set with 158 patterns was used for this testing: it has an exponential distribution and the least preferred model states are present. Looking at the fully specified premises, "1, -1, -1, -1," and "1, 1, -1, 1, 1", it can be seen that the machine was adequately trained across the whole distribution (a most preferred state, decimal 16 - output 97% and a least preferred state, decimal 27 - output 89%).

Despite the failings of testing against the neutral premise, selection of model states from the specific premises was remarkably appropriate. For example consider premise "0, -1, 0, -1, 1"; this premise selects states over a range of the most preferred models. The selected states are correct, their ranking is correct and the magnitude of the separation between states is appropriate. Similarly premises "0, -1, 0, 0, 1" and "1, -1, 0, 0, 1" select a range of states, including the least preferred models; again these states are correctly selected and ranked with appropriate separation. Only premise "0, -1, 0, 0, 0" causes difficulty; it is quite close to the neutral premise and requires 16 model states to be retrieved. Still this result is acceptable, but with a narrow margin between each of the models.

#### 3.6 6-Atom Micro-World

The results of testing against the neutral premise in the 6-atom micro-world were mixed (Figures 21 & 22 - only two of the training sets are shown because of their size). The average error per state on the worst training set with 332 patterns was just over 1% and on the best training set with 180 patterns was a remarkable  $0.3\% \pm 0.02$ . However, on training sets where a large majority of patterns are expected to return 0, the average error per pattern will be 'diluted'. As for the 5-atom world, the machine underestimated the probability of the most preferred model states on all the training sets. Moreover, on the training sets with least preferred patterns present, the small percentage separation between ranking levels became blurred or even overlapped.

Again for completeness, it was decided that 12 specific premises would be tested (Figures 23 & 24). The training set with 232 patterns was used for this testing: it has an arithmetic distribution and the least preferred model states are present. Looking at the fully specified premises, "1, -1, -1, -1, -1, -1" and "-1, 1, 1, 1, 1, 1", it can be seen that the machine was adequately trained across the whole distribution (a most preferred state, decimal 31 - output 96% and a least preferred state, decimal 27 - output 93%).

Despite the shortcomings of the testing with the neutral premise, selection of model states from the more specific premises was accurate. For premises with less than three uncertain atoms the machine correctly selects and ranks models states, with a degree of separation appropriate for the ranked interpretation. Model selection only becomes marginal with three uncertainties, for example "0, -1, 0, -1, 0, 1". For premise "0, -1, 0, -1, -1, 0" the selection of the most preferred states is overlapped: (32), (33), (0) and (1), but in this situation the machine is 'performing a tie break' based on the atomic probability of dependent variables.

We did not test the Boltzmann machine representation beyond the 6-atom world: considering its stochastic error of approximately 2% per state and that the next level of complexity (7-atoms/preference levels) comprises 128 states. The Boltzmann machine is known to scale poorly from small to larger tasks but this could perhaps be addressed by a longer, slower annealing and training process.

## 4 Multilayer Perceptron (MLP)

The MLP was chosen as a typical candidate feed-forward network, to ascertain whether this most common type of neural network could represent a probabilistic distribution over the output states.

#### 4.1 Implementation

The MLP architecture for our representation consisted of input nodes, one for each atomic variable (n) in the micro-world, a hidden layer and output nodes locally coded, one for each of the world states  $(2^n)$ . Biases were included in both the hidden and output layers. The implementation can be seen as a function converting binary to decimal numbers. The implementation was first separately verified with a flat distribution over all 16 training patterns in the 4-atom micro-world and tuned with regard to: number of hidden nodes, learning rate (typically 0.4) and momentum (typically 0.8). The final numbers of hidden nodes were: 6 in the 4-atom world and 10 in the 5-atom world. Verification on previously seen atomic training patterns gave single output activations on the corresponding decimal state of over 0.9 and activations of under 0.1 on the others.

#### 4.2 *Results in General*

The MLP was only tested in the 4-atom and 5-atom micro-worlds. Machines were trained with the proportional set of patterns based on the ranked interpretation derived from the default rules of the micro-world. The training and testing followed exactly in the fashion of the Boltzmann machines. The results for testing of the MLP representation are given in Appendix C.

#### 4.3 4-Atom Micro-World

The results of testing against the neutral premise in the 4-atom micro-world were uniformly unsatisfactory (Figures 25 & 26). The average error per state on the worst training set with 96 patterns was over 10% and on the best training set with 90 patterns was over 8%. Examination of any of the individual machine runs or average results confirms that the network is not representing the input distribution.

There is however, a subtle relationship between the expected distribution and the machine output. The machine weights are maximally trained to the states, which are the least frequent input patterns. These maximally trained weights have a predominate effect on the network output because of the low overall activation of the network. In the case of training sets 88-0 and 90-0 the intermediate preference states are least frequent (the least preferred states are absent). The architecture and learning algorithm of the MLP are not designed for the task of 'recalling' the training set distribution.

The results for testing against 28 specific premises are shown in Figures 27 - 30). The training set with 98 patterns was used. Looking at the fully specified premises; "-1, -1, -1", "-1, -1, 1", "1, -1, -1" and "1, -1, -1, 1", it can be seen that the machine was adequately trained across the whole distribution (a most preferred state, decimal 0 - output 99% and a least preferred state, decimal 6 - output 97%).

The vast majority of results for the individual specific premises indicate that the machine can select model states, but not in the correct preference ranking and not with an appropriate degree of separation between the levels of preference. For most of the premises where selection of multiple levels is required, the machine often

outputs a uniform distribution across all levels of preference, for example premises "-1, 0, 0, 1" and "-1, -1, 0, 0". The more uncertainty in the premise, the poorer is the representation of the input distribution.

Where the MLP is given almost complete information in the premise, Figure 30, it is able to appropriately rank two model states at different levels.

#### 4.4 5-Atom Micro-World

The results of testing against the neutral premise in the 5-atom micro-world were uniformly unsatisfactory (Figures 31- 34). The average error per state on the worst training set with 142 patterns was close to 6% and on the best training set with 136 patterns was over 4%. This may seem reasonable but examination of any of the individual machine runs or average results confirms that the network is not representing the input distribution.

The same subtle relationship between the expected distribution and the machine output applies in this micro-world as in the 4-atom world. The machine weights are maximally trained to the states, which are the least frequent input patterns. The architecture and learning algorithm of the MLP are not designed for this task.

The results for the individual specific premises (Figures 35 - 37) indicated as in the 4-atom world, that although the machine can select correct model states: the preference ranking and degree of separation between the levels of preference were incorrect. Again, where selection of multiple levels of preference was required, the machine would often output a uniform distribution. The more uncertainty in the premise, the poorer was the representation of the input distribution. This can be seen in Figure 37 where two bits of the premise are uncertain or for example in premise "0, -1, 0, 0, 1" where three bits of the premise are uncertain. Often one of the least preferred model states had the highest output. Even where the premise was almost fully specified, for example "1, -1, -1, 0" the MLP was not able to place two model states appropriately at the same level.

The MLP representation was not tested beyond the 5-atom (preference level) environment as we felt its performance had already been unsatisfactory at both the previous levels.

## Appendix A: Micro-Worlds

Fan, Hea	iter, Window			8 Poss	ible state	s of th	e world		
	f = fan on, h	= heat	ter on, w	= winc	low open				
Default R	ules:								
	Energy cons	ervatio	on, usuall	y ¬F ar	nd ¬H, W	has no	o cost		
	Warm enviro	onmer	nt. F more	e comr	non than	н			
	Consistency	F -> V	, V H -> ¬V	V					
	consistency		,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	•					
			Exponen	tial			Arithmet	ic	
Ranked Ir	ntrepretation	#	Exponen %	tial #	%	#	Arithmet %	ic #	%
Ranked Ir -1-1-1(0)	ntrepretation -1-11(1)	#	Exponen % 40.0%	tial # 16	<i>%</i> 36.4%	#	Arithmet % 33.3%	ic # 10	<i>%</i> 29.4%
<b>Ranked I</b> -1-1-1(0)	ntrepretation -1-11(1)	# 16	Exponen % 40.0%	tial # 16	% 36.4%	#	Arithmet % 33.3%	ic # 10	<i>%</i> 29.4%
Ranked Ir -1-1-1(0) 1-1-1(4)	1-11(1)	# 16 4	Exponen % 40.0% 10.0%	tial # 16 4	% 36.4% 9.1%	# 10 5	Arithmet % 33.3% 16.7%	ic # 10 5	% 29.4% 14.7%
<b>Ranked Ir</b> -1-1-1(0) 1-1-1(4)	ntrepretation -1-11(1) 1-11(5)	# 16 4	Exponen % 40.0% 10.0%	tial # 16 4	% 36.4% 9.1%	# 10 5	Arithmet % 33.3% 16.7%	ic # 10 5	% 29.4% 14.7%
<b>Ranked Ir</b> -1-1-1(0) 1-1-1(4) -11-1(2)	ntrepretation -1-11(1) 1-11(5) -111(3)	# 16 4 0	Exponent % 40.0% 10.0% 0.0%	tial # 16 4 1	% 36.4% 9.1% 2.3%	# 10 5 0	Arithmet % 33.3% 16.7% 0.0%	ic # 10 5 1	% 29.4% 14.7% 2.9%
Ranked Ir -1-1-1(0) 1-1-1(4) -11-1(2) 11-1(6)	ntrepretation -1-11(1) 1-11(5) -111(3) 111(7)	# 16 4 0	Exponent % 40.0% 10.0% 0.0%	tial # 16 4 1	% 36.4% 9.1% 2.3%	# 10 5 0	Arithmet % 33.3% 16.7% 0.0%	ic # 10 5 1	% 29.4% 14.7% 2.9%

*Figure 1.* Details of the 3-atom micro-world. Ranked interpretation and the distribution of patterns in the training sets are shown.

Light, Fan	, Heater, W	/indow			16 Poss	ible sta	ates of th	e worl	d		
	l = light on,	f = fan on, ł	n = heater on,	w = wii	ndow ope	en					
Default Rul	es:										
	Energy con	servation, u	sually ¬F and -	H, Wa	and L hav	e no co	ost				
	Warm envi	ronment, Fi	more common	than H	4						
	Consistency	, F -> W, H	-> ¬W								
	L is indeper	ndent									
					Exponen	tial			Arithmet	ic	
Ranked Int	repretation			#	%	#	%	#	%	#	%
-1-1-1-1(0)	-1-1-11(1)	1-1-1(8)	1-1-11(9)	16	18.2%	16	16.7%	15	16.7%	15	15.3%
-11-11(5)	11-11(13)			8	9.1%	8	8.3%	10	11.1%	10	10.2%
-1-11-1(2)	1-11-1(10)			4	4.5%	4	4.2%	5	5.6%	5	5.1%
-1-111(3)	-11-1-1(4)	-111-1(6)	-1111(7)	0	0.0%	1	1.0%	0	0.0%	1	1.0%
$1_111(11)$	11-1-1(12)	111-1(14)	1111(15)								
	/		• •								

*Figure 2.* Details of the 4-atom micro-world. Ranked interpretation and the distribution of patterns in the training sets are shown.

l = light on, f = fan on, a = aircon on, h = h	neater	on, w = w	vindow	open /				
Default Rules:								
L independent and usually on								
Energy conservation, usually ¬A, ¬F and ·	-H, W a	and L hav	e no c	ost				
Warm environment, A and F more comm	non tha	n H						
Consistency, A -> W, F -> W, H -> $\neg$ W								
		_		1				
Production and the		Expone	ntial	04		Arithmet	ic "	04
	#	%	#	%	#	%	#	%
1-1-1-1-1(16) 1-1-1-11(17)	32	23.5%	32	20.3%	20	16.7%	20	14.1%
-1-1-1-1(0) -1-1-1-11(1)	16	11.8%	16	10.1%	15	12.5%	15	10.6%
-1-11-11(5) -11-1-11(9) 1-11-11(21) 11-1-11(25)	8	5.9%	8	5.1%	10	8.3%	10	7.0%
1 1 11 1/2) 1 1 11 1/10)		2.0%		2 50/	-	4.20/	-	2 50/
-1-1-11-1(2) 1-1-11-1(18)	4	2.9%	4	2.5%	5	4.2%	5	3.5%
All other states 22	0	0.0%	1	0.6%	0	0.0%	1	0.7%
(3), (4), (6) - (8),								
(10) - (15), (19), (20),								
(22) - (24), (26) - (31)								

*Figure 3.* Details of the 5-atom micro-world. Ranked interpretation and the distribution of patterns in the training sets are shown.

Light, Fan, Airconditioning, Heater, Open Fire, Window		fine lite .	64 Pos	sible stat	tes of t	the world		
I = light on, t = tan on, a = aircon on, h = heater Default Rules: L independent and usually on Energy conservation, usually ¬O, ¬A, ¬F and ¬H Warm environment, A and F more common the	on, o : I, W an an O ai	= fire lit, v id L have nd H	w = wi no cos	ndow ope	en			
Consistency, A -> W, F -> W, O -> ¬W, H -> ¬W								
		Exponen	tial			Arithmet	ic	
Ranked Intrepretation	#	%	#	%	#	%	#	%
1-1-1-1-1(32) 1-1-1-11(33)	64	22.9%	64	19.3%	25	13.9%	25	10.8%
1-1-1-1-1(0) -1-1-1-11(1)	32	11.4%	32	9.6%	20	11.1%	20	8.6%
.1-11-1-11(9) -11-1-1-11(17) 1-11-1-11(41) 11-1-1-11(49)	16	5.7%	16	4.8%	15	8.3%	15	6.5%
1-1-11-1-1(4) 1-1-11-1-1(36)	8	2.9%	8	2.4%	10	5.6%	10	4.3%
-1-1-11-1(2) 1-1-1-11-1(34)	4	1.4%	4	1.2%	5	2.8%	5	2.2%
All other states 52	0	0.0%	1	0.3%	0	0.0%	1	0.4%
3), (5) - (8), (10) - (16) 18) - (31), (35), (37) - (40), (42), (48), (52), (53)								

*Figure 4.* Details of the 6-atom micro-world. Ranked interpretation and the distribution of patterns in the training sets are shown.

## **Appendix B: Blotzmann Machine Results**



*Figure 5.* Results for the Boltzmann machine in the 3-atom micro-world, tested against the neutral premise. States are given in decimal format.

Ranked Interpret.	State	Expected %	Samples	Output %	Samples	Output %	Samples	Output %	Samples	Output %	Samples	Output %	Samples	Output %	]
(0), (1)	0	33.3%	19146	32%	20541	34%	19558	33%	20262	34%	19393	32%	98900	33.0%	1
	1	33.3%	19843	33%	18646	31%	18842	31%	19381	32%	19579	33%	96291	32.1%	
(4), (5)	2	0.0%	77	0%	90	0%	70	0%	63	0%	64	0%	364	0.1%	
	3	0.0%	82	0%	82	0%	88	0%	82	0%	79	0%	413	0.1%	
(2), (3), (6), (7)	4	16.7%	10233	17%	10073	17%	10988	18%	10392	17%	10811	18%	52497	17.5%	
	5	16.7%	10440	17%	10392	17%	10286	17%	9636	16%	9911	17%	50665	16.9%	
	6	0.0%	88	0%	85	0%	81	0%	85	0%	82	0%	421	0.1%	
	7	0.0%	91	0%	91	0%	87	0%	99	0%	81	0%	449	0.1%	
	Accument			3 30/						3 30/					
	Accumu	atea error =		3.2%		4.4%		5.2%		3.3%		3.7%		3.1%	± 0.85
A	vg. erro	atea error = r per state =		3.2% 0.4%		4.4% 0.6%		5.2% 0.7%		3.3% 0.4%		3.7% 0.5%		3.1% 0.4%	± 0.85 ± 0.11
Å	Trainii	ng Set: 34-1		3.2% 0.4%		4.4% 0.6%		5.2% 0.7%		3.3% 0.4%		3.7% 0.5%		3.1% 0.4%	± 0.85 ± 0.11
A Ranked Interpret.	Trainii State	ng Set: 34-1 Expected %	Samples	3.2% 0.4% Output %	Samples	4.4% 0.6% Output %	Samples	5.2% 0.7% Output %	Samples	3.3% 0.4% Output %	Samples	3.7% 0.5% Output %	Samples	3.1% 0.4% Output %	± 0.85 ± 0.11
Ranked Interpret. (0), (1)	Trainin State	natea error = r per state = ng Set: 34-1 Expected % 29.4%	Samples 13292	3.2% 0.4% Output % 22%	Samples 13786	4.4% 0.6% Output % 23%	Samples 15022	<b>5.2%</b> <b>0.7%</b> Output % 25%	Samples 14518	0.4% 0.4% Output % 24%	Samples 14415	3.7% 0.5% Output % 24%	Samples 71033	3.1% 0.4% Output % 23.7%	± 0.85 ± 0.11
Ranked Interpret. (0), (1)	Trainii State 0 1	atea error = r per state = ng Set: 34-1 Expected % 29.4% 29.4%	Samples 13292 18339	3.2% 0.4% Output % 22% 31%	Samples 13786 17681	4.4% 0.6% Output % 23% 29%	Samples 15022 17560	5.2% 0.7% Output % 25% 29%	Samples 14518 17384	0.4% 0.4% 0utput % 24% 29%	Samples 14415 17443	3.7% 0.5% Output % 24% 29%	Samples 71033 88407	3.1% 0.4% Output % 23.7% 29.5%	± 0.85 ± 0.11
A Ranked Interpret. (0), (1) (4), (5)	Trainin State 0 1 2	atea error = r per state = ng Set: 34-1 Expected % 29.4% 29.4% 2.9%	Samples 13292 18339 1791	3.2% 0.4% Output % 22% 31% 3%	Samples 13786 17681 1836	4.4% 0.6% Output % 23% 29% 3%	Samples 15022 17560 1839	5.2% 0.7% Output % 25% 29% 3%	Samples 14518 17384 1867	3.3% 0.4% Output % 24% 29% 3%	Samples 14415 17443 1715	3.7% 0.5% Output % 24% 29% 3%	Samples 71033 88407 9048	3.1% 0.4% Output % 23.7% 29.5% 3.0%	± 0.85 ± 0.11
Ranked Interpret. (0), (1) (4), (5)	Trainii State 0 1 2 3	atea error = r per state = ng Set: 34-1 Expected % 29.4% 2.9% 2.9% 2.9%	Samples 13292 18339 1791 3070	3.2% 0.4% Output % 22% 31% 3% 5%	Samples 13786 17681 1836 3123	4.4% 0.6% 0utput % 23% 29% 3% 5%	Samples 15022 17560 1839 3269	5.2% 0.7% 0.7% 25% 29% 3% 5%	Samples 14518 17384 1867 3006	3.3% 0.4% 0utput % 24% 29% 3% 5%	Samples 14415 17443 1715 3109	3.7% 0.5% Output % 24% 29% 3% 5%	Samples 71033 88407 9048 15577	3.1% 0.4% 23.7% 29.5% 3.0% 5.2%	± 0.85 ± 0.11
Ranked Interpret. (0), (1) (4), (5) (2), (3), (6), (7)	Trainin State 0 1 2 3 4	area error = r per state = ng Set: 34-1 Expected % 29.4% 2.9% 2.9% 2.9% 14.7%	Samples 13292 18339 1791 3070 9196	3.2% 0.4% Output % 22% 31% 3% 5% 15%	Samples 13786 17681 1836 3123 9566	4.4% 0.6% 23% 29% 3% 5% 16%	Samples 15022 17560 1839 3269 8596	5.2% 0.7% 0.7% 25% 29% 3% 5% 14%	Samples 14518 17384 1867 3006 8598	3.3% 0.4% 24% 29% 3% 5% 14%	Samples 14415 17443 1715 3109 8742	3.7% 0.5% Output % 24% 29% 3% 5% 15%	Samples 71033 88407 9048 15577 44698	3.1% 0.4% 23.7% 29.5% 3.0% 5.2% 14.9%	± 0.85 ± 0.11
Ranked Interpret. (0), (1) (4), (5) (2), (3), (6), (7)	Trainin State 0 1 2 3 4 5	area error = r per state = ng Set: 34-1 Expected % 29.4% 2.9% 2.9% 14.7% 14.7%	Samples 13292 18339 1791 3070 9196 9465	3.2% 0.4% 0utput % 22% 31% 3% 5% 15% 16%	Samples 13786 17681 1836 3123 9566 9034	4.4% 0.6% 23% 29% 3% 5% 16% 15%	Samples 15022 17560 1839 3269 8596 8596 8700	5.2% 0.7% 25% 29% 3% 5% 14% 15%	Samples 14518 17384 1867 3006 8598 9393	3.3% 0.4% 24% 29% 3% 5% 14% 16%	Samples 14415 17443 1715 3109 8742 9379	3.7% 0.5% 0.5% 24% 29% 3% 5% 15% 16%	Samples 71033 88407 9048 15577 44698 45971	3.1% 0.4% 23.7% 29.5% 3.0% 5.2% 14.9% 15.3%	± 0.85 ± 0.11
Ranked Interpret. (0), (1) (4), (5) (2), (3), (6), (7)	Trainit State 0 1 2 3 4 5 6	area error = r per state = ng Set: 34-1 Expected % 29.4% 2.9% 2.9% 14.7% 14.7% 2.9%	Samples 13292 18339 1791 3070 9196 9465 1815	3.2% 0.4% 22% 31% 3% 5% 15% 16% 3%	Samples 13786 17681 1836 3123 9566 9034 2049	4.4% 0.6% 23% 29% 3% 5% 16% 15% 3%	Samples 15022 17560 1839 3269 8596 8700 1905	5.2% 0.7% 25% 29% 3% 5% 14% 15% 3%	Samples 14518 17384 1867 3006 8598 9393 2002	3.3% 0.4% 24% 29% 3% 5% 14% 16% 3%	Samples 14415 17443 1715 3109 8742 9379 2127	3.7% 0.5% 24% 29% 3% 5% 15% 16% 4%	Samples 71033 88407 9048 15577 44698 45971 9898	3.1% 0.4% 23.7% 29.5% 3.0% 5.2% 14.9% 15.3% 3.3%	± 0.85 ± 0.11
Ranked Interpret. (0), (1) (4), (5) (2), (3), (6), (7)	Trainii State 0 1 2 3 4 5 6 7	area error = r per state = ng Set: 34-1 Expected % 29.4% 2.9% 2.9% 14.7% 14.7% 2.9% 2.9% 2.9%	Samples 13292 18339 1791 3070 9196 9465 1815 3032	3.2% 0.4% 22% 31% 3% 5% 15% 16% 3% 5%	Samples 13786 17681 1836 3123 9566 9034 2049 2925	4.4% 0.6% 23% 29% 3% 5% 16% 15% 3% 5%	Samples 15022 17560 1839 3269 8596 8700 1905 3109	5.2% 0.7% 25% 29% 3% 5% 14% 15% 3% 5%	Samples 14518 17384 1867 3006 8598 9393 2002 3232	3.3% 0.4% 24% 29% 3% 5% 14% 16% 3% 5%	Samples 14415 17443 1715 3109 8742 9379 2127 3070	3.7% 0.5% Output % 24% 29% 3% 5% 15% 16% 4% 5%	Samples 71033 88407 9048 15577 44698 45971 9898 15368	3.1% 0.4% 0.4% 23.7% 29.5% 3.0% 5.2% 14.9% 15.3% 3.3% 5.1%	± 0.85 ± 0.11
Ranked Interpret. (0), (1) (4), (5) (2), (3), (6), (7)	Trainii State 0 1 2 3 4 5 6 7 Accumul	atea error = r per state = ng Set: 34-1 Expected % 29.4% 2.9% 2.9% 14.7% 14.7% 2.9% 2.9% ated error =	Samples 13292 18339 1791 3070 9196 9465 1815 3032	3.2% 0.4% 22% 31% 5% 15% 16% 3% 5% 14.7%	Samples 13786 17681 1836 3123 9566 9034 2049 2925	4.4% 0.6% 23% 29% 29% 3% 5% 16% 15% 3% 5% 15% 3% 5% 13.0%	Samples 15022 17560 1839 3269 8596 8700 1905 3109	5.2% 0.7% 25% 29% 3% 5% 14% 15% 3% 5% 10.3%	Samples 14518 17384 1867 3006 8598 9393 2002 3232	3.3% 0.4% 24% 22% 3% 5% 14% 16% 3% 5% 12.2%	Samples 14415 17443 1715 3109 8742 9379 2127 3070	3.7% 0.5% 24% 29% 3% 5% 15% 16% 4% 5% 12.0%	Samples 71033 88407 9048 15577 44698 45971 9898 15368	3.1% 0.4% 23.7% 29.5% 3.0% 5.2% 14.9% 15.3% 3.3% 5.1% 11.6%	± 0.85 ± 0.11

*Figure 6.* Results for the Boltzmann machine in the 3-atom micro-world, tested against the neutral premise. States are given in decimal format.



*Figure 7.* Results for the Boltzmann machine in the 3-atom micro-world, tested against specific premises. States are given in decimal format.

Stdev	28																	± 1.12	± 0.07	2																	1
Avg	Output 9	19.0%	18.1%	5.6%	0.1%	0.1%	7.1%	0.2%	0.1%	19.2%	18.1%	5.4%	0.1%	0.1%	6.8%	0.1%	0.1%	9.1%	0.6%	Output 9	13.8%	14.5%	5.5%	2.8%	3.2%	6.4%	1.9%	2.0%	13.8%	14.4%	4.9%	3.0%	3.3%	6.6%	1.8%	2.0%	
	Samples	56904	54333	16699	190	176	21379	476	420	57518	54169	16282	173	191	20255	447	388			Samples	41513	43376	16438	8524	9648	19346	5798	6077	41306	43076	14797	8987	9813	19909	5506	5886	
	Output %	18.3%	19.1%	5.8%	0.1%	0.1%	7.6%	0.1%	0.1%	19.2%	17.8%	5.8%	0.0%	0.1%	5.8%	0.1%	0.1%	10.7%	0.7%	Output %	14.2%	15.1%	5.5%	3.1%	3.1%	6.1%	1.8%	1.7%	13.9%	14.2%	4.9%	2.9%	3.3%	6.3%	1.9%	2.1%	]
	Samples	10953	11434	3458	45	36	4548	70	82	11520	10693	3498	23	38	3451	64	87			Samples	8497	9064	3320	1838	1867	3672	1108	1018	8330	8533	2916	1721	1974	3771	1137	1234	
	Output %	18.7%	17.7%	5.1%	0.1%	0.1%	6.6%	0.3%	0.2%	19.8%	19.4%	4.9%	0.1%	0.1%	6.7%	0.2%	0.1%	10.9%	0.7%	Output %	13.3%	14.7%	5.9%	2.9%	3.1%	6.8%	1.8%	2.1%	13.8%	14.4%	5.2%	2.8%	3.4%	6.3%	1.7%	1.8%	
	Samples	11241	10611	3057	37	36	3981	158	66	11907	11655	2924	37	52	4042	92	71			Samples	7964	8834	3519	1749	1876	4066	1081	1279	8258	8617	3110	1696	2069	3774	1024	1084	
	Output %	19.1%	17.8%	5.7%	0.0%	0.1%	7.0%	0.1%	0.2%	19.0%	17.8%	5.4%	0.0%	0.1%	7.4%	0.1%	0.2%	9.2%	0.6%	Output %	14.5%	13.7%	4.6%	2.6%	3.7%	6.5%	2.1%	2.2%	13.7%	13.6%	4.6%	3.0%	3.7%	7.4%	2.0%	2.1%	
	Samples	11439	10698	3390	26	39	4197	59	109	11428	10675	3263	29	60	4419	79	06			Samples	8709	8235	2769	1548	2219	3883	1278	1331	8212	8138	2732	1792	2222	4451	1223	1258	
	Output %	19.7%	17.6%	6.0%	0.1%	0.0%	7.0%	0.1%	0.1%	19.0%	17.2%	5.5%	0.1%	%0.0	7.2%	0.2%	0.1%	11.2%	0.7%	Output %	13.2%	13.8%	5.9%	2.8%	2.8%	6.9%	2.0%	1.9%	14.3%	14.5%	5.2%	3.0%	2.9%	7.0%	1.8%	2.0%	
	Samples	11805	10585	3626	50	28	4214	86	65	11373	10303	3284	35	21	4313	125	87			Samples	7920	8305	3528	1682	1692	4122	1187	1158	8562	8714	3107	1796	1768	4221	1058	1180	
	Output %	19.1%	18.3%	5.3%	0.1%	0.1%	7.4%	0.2%	0.1%	18.8%	18.1%	5.5%	0.1%	%0.0	6.7%	0.1%	0.1%	8.6%	0.5%	Output %	14.0%	14.9%	5.5%	2.8%	3.3%	6.0%	1.9%	2.2%	13.2%	15.1%	4.9%	3.3%	3.0%	6.2%	1.8%	1.9%	
	Samples	11466	11005	3168	32	37	4439	103	65	11290	10843	3313	49	20	4030	87	53			Samples	8423	8938	3302	1707	1994	3603	1144	1291	7944	9074	2932	1982	1780	3692	1064	1130	
0-00 -1ac	Expected %	18.1%	18.1%	4.5%	0.0%	0.0%	9.1%	0.0%	0.0%	18.1%	18.1%	4.5%	0.0%	0.0%	9.1%	0.0%	0.0%	ated error =	r per state =	Expected %	16.7%	16.7%	4.2%	1.0%	1.0%	8.3%	1.0%	1.0%	16.7%	16.7%	4.2%	1.0%	1.0%	8.3%	1.0%	1.0%	
2 IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII	State	0	1	2	e	4	5	9	7	80	6	10	11	12	13	14	15	Accumula	Avg. erroi	State	0	1	2	ß	4	2	9	7	80	6	10	11	12	13	14	15	
	nked interpret.	, (1), (8), (9)		, (13)		, (10)		, (4), (6), (7),	), (12), (14), (15)											ked Interpret.	(1), (8), (9)		, (13)		(10)		(4), (6), (7),	), (12), (14), (15)									2

*Figure 8.* Results for the Boltzmann machine in the 4-atom micro-world, tested against the neutral premise. States are given in decimal format.

anked Interpret.	State	Expected %	Samples	Output %	Samples	Output %	Samples	Output %	Samples	Output %	Samples	Output %	Samples	Output %	
(1), (1), (8), (9)	0	16.7%	10582	17.6%	10478	17.5%	10831	18.1%	9868	16.4%	10613	17.7%	52372	17.5%	
	1	16.7%	10956	18.3%	10941	18.2%	11211	18.7%	11198	18.7%	11258	18.8%	55564	18.5%	
(), (13)	2	5.6%	3512	5.9%	2995	5.0%	3320	5.5%	3100	5.2%	2987	5.0%	15914	5.3%	
	e	0.0%	39	0.1%	29	0.0%	44	0.1%	37	0.1%	32	0.1%	181	0.1%	
(), (10)	4	0.0%	30	0.1%	33	0.1%	24	%0.0	32	0.1%	20	0.0%	139	%0.0	
	S	11.1%	5232	8.7%	5145	8.6%	5100	8.5%	5442	9.1%	5406	9.0%	26325	8.8%	
(), (4), (6), (7),	9	0.0%	73	0.1%	50	0.1%	57	0.1%	79	0.1%	53	0.1%	312	0.1%	
1), (12), (14), (15)	7	0.0%	63	0.1%	98	0.2%	116	0.2%	74	0.1%	111	0.2%	462	0.2%	
	8	16.7%	10292	17.2%	10651	17.8%	10388	17.3%	10299	17.2%	10600	17.7%	52230	17.4%	
	6	16.7%	10522	17.5%	11181	18.6%	10645	17.7%	11196	18.7%	10882	18.1%	54426	18.1%	
	10	5.6%	3146	5.2%	3078	5.1%	2903	4.8%	3114	5.2%	2513	4.2%	14754	4.9%	
	11	0.0%	35	0.1%	33	0.1%	33	0.1%	41	0.1%	39	0.1%	181	0.1%	
	12	0.0%	32	0.1%	26	0.0%	36	0.1%	33	0.1%	23	0.0%	150	0.1%	
	13	11.1%	5317	8.9%	5093	8.5%	5098	8.5%	5337	8.9%	5299	8.8%	26144	8.7%	
	14	0.0%	63	0.1%	63	0.1%	80	0.1%	60	0.1%	35	0.1%	301	0.1%	
	15	0.0%	106	0.2%	106	0.2%	114	0.2%	<del>00</del>	0.2%	129	0.2%	545	0.2%	
	Accumul	ated error =		9.8%		12.2%		11.9%		10.5%		12.6%		11.2%	± 1.21
	Avg. erro	r per state =		0.6%		0.8%		0.7%		0.7%		0.8%		0.7%	± 0.0
	Training	Set: 98-1													
anked interpret.	State	Expected %	Samples	Output %	Samples	Output %	Samples	Output %	Samples	Output %	Samples	Output %	Samples	Output %	
(1), (1), (8), (9)	0	15.3%	8001	13.3%	8109	13.5%	7299	12.2%	7766	12.9%	7497	12.5%	38672	12.9%	
	1	15.3%	7268	12.1%	8124	13.5%	7854	13.1%	8368	13.9%	7929	13.2%	39543	13.2%	
(), (13)	2	5.1%	3044	5.1%	2756	4.6%	3238	5.4%	3105	5.2%	3093	5.2%	15236	5.1%	
	ß	1.0%	1451	2.4%	1679	2.8%	1634	2.7%	1826	3.0%	1732	2.9%	8322	2.8%	
(), (10)	4	1.0%	2089	3.5%	2226	3.7%	2186	3.6%	2045	3.4%	2118	3.5%	10664	3.6%	
	S	10.2%	4806	8.0%	4783	8.0%	5012	8.4%	4683	7.8%	4942	8.2%	24226	8.1%	
(), (4), (6), (7),	9	1.0%	1120	1.9%	1179	2.0%	1097	1.8%	1051	1.8%	1147	1.9%	5594	1.9%	
1), (12), (14), (15)	7	1.0%	1309	2.2%	1271	2.1%	1406	2.3%	1455	2.4%	1188	2.0%	6629	2.2%	
	80	15.3%	8420	14.0%	8062	13.4%	8470	14.1%	7574	12.6%	7917	13.2%	40443	13.5%	
	6	15.3%	8372	14.0%	8036	13.4%	8322	13.9%	8228	13.7%	8458	14.1%	41416	13.8%	
	10	5.1%	3047	5.1%	2985	5.0%	3154	5.3%	3084	5.1%	2961	4.9%	15231	5.1%	
	11	1.0%	1498	2.5%	1685	2.8%	1684	2.8%	1686	2.8%	1592	2.7%	8145	2.7%	
	12	1.0%	2268	3.8%	2210	3.7%	2042	3.4%	2231	3.7%	2271	3.8%	11022	3.7%	
	13	10.2%	4871	8.1%	4507	7.5%	4217	7.0%	4552	7.6%	4794	8.0%	22941	7.6%	
	14	1.0%	1005	1.7%	1218	2.0%	1103	1.8%	1018	1.7%	1148	1.9%	5492	1.8%	
	15	1.0%	1431	2.4%	1170	2.0%	1282	2.1%	1328	2.2%	1213	2.0%	6424	2.1%	
	Accumul	ated error =		24.4%		25.9%		26.2%		26.2%		25.3%		25.3%	± 0.7

*Figure 9.* Results for the Boltzmann machine in the 4-atom micro-world, tested against the neutral premise. States are given in decimal format.



*Figure 10.* Results for the Boltzmann machine in the 4-atom micro-world, tested against specific premises. States are given in decimal format.



*Figure 11.* Results for the Boltzmann machine in the 4-atom micro-world, tested against specific premises. States are given in decimal format.



*Figure 12.* Results for the Boltzmann machine in the 4-atom micro-world, tested against specific premises. States are given in decimal format.



*Figure 13.* Results for the Boltzmann machine in the 4-atom micro-world, tested against specific premises. States are given in decimal format.

Junev																																		
RVB	Output %	7.1%	16.6%	3.6%	0.0%	%0.0	8.3%	0.3%	0.0%	0.1%	8.3%	0.3%	0.0%	0.0%	1.4%	%0.0	%0'0	9.4%	21.8%	4.0%	0.0%	%0.0	8.5%	0.2%	%0.0	0.1%	8.3%	0.2%	0.0%	%0.0	1.4%	0.0%	%0.0	100 01
	Samples	21245	49911	10812	75	141	24998	789	73	156	24982	756	68	2	4076	7	57	28127	65498	11900	109	136	25633	642	84	178	24819	563	65	1	4064	7	26	000000
	Output %	7.2%	16.2%	3.7%	0.0%	0.0%	8.6%	0.3%	0.0%	0.1%	8.0%	0.3%	0.0%	0.0%	1.4%	0.0%	0.1%	9.1%	22.6%	3.3%	0.0%	0.0%	9.0%	0.3%	0.0%	0.0%	8.0%	0.2%	0.0%	0.0%	1.4%	0.0%	0.0%	
	Samples	4302	9718	2207	17	27	5167	165	13	33	4801	180	12	0	841	4	36	5485	13550	1982	26	29	5384	174	18	29	4820	114	13	0	850	1	2	00005
	Output %	7.1%	16.1%	3.8%	0.0%	0.0%	8.4%	0.2%	0.0%	0.1%	8.4%	0.2%	0.0%	0.0%	1.5%	%0.0	%0.0	8.9%	21.6%	3.9%	%0.0	0.1%	8.0%	0.2%	%0.0	0.1%	9.0%	0.2%	0.0%	%0.0	1.3%	0.0%	0.0%	
	Samples	4273	9636	2260	14	24	5037	141	19	33	5011	103	12	0	896	0	5	5312	12936	2356	20	32	5401	137	23	32	5417	95	6	0	763	e	0	00000
	Output %	7.7%	17.7%	3.4%	0.0%	0.1%	7.6%	0.2%	0.0%	0.1%	8.0%	0.3%	0.0%	0.0%	%6.0	0.0%	%0.0	9.6%	23.2%	3.8%	0.0%	0.0%	8.4%	0.2%	0.0%	0.1%	7.4%	0.2%	0.0%	0.0%	1.0%	0.0%	0.0%	
	Samples	4617	10622	2059	14	43	4548	134	17	37	4806	183	22	0	516	1	4	5738	13897	2298	14	27	5048	117	13	33	4469	110	6	0	595	0	6	0000
	Output %	6.9%	16.6%	3.6%	%0.0	%0.0	8.2%	0.3%	0.0%	%0.0	8.6%	0.3%	%0.0	%0.0	1.3%	%0.0	%0.0	9.3%	21.6%	4.4%	%0.0	%0.0	8.5%	0.1%	%0.0	0.1%	8.6%	0.2%	%0.0	%0.0	1.3%	0.0%	0.0%	
	Samples	4110	9962	2131	15	25	4909	173	18	27	5134	175	15	Ļ	752	1	7	5582	12939	2620	21	27	5075	83	16	61	5187	133	16	0	782	0	3	00000
	Output %	6.6%	16.6%	3.6%	0.0%	0.0%	8.9%	0.3%	0.0%	0.0%	8.7%	0.2%	%0.0	0.0%	1.8%	%0.0	%0.0	10.0%	20.3%	4.4%	0.0%	%0.0	7.9%	0.2%	%0.0	%0.0	8.2%	0.2%	0.0%	0.0%	1.8%	0.0%	0.0%	100 11
	Samples	3943	9973	2155	15	22	5337	176	9	26	5230	115	7	1	1071	1	5	6010	12176	2644	28	21	4725	131	14	23	4926	111	18	1	1074	'n	12	0000
	Expected %	11.8%	11.8%	2.9%	0.0%	0.0%	5.9%	0.0%	0.0%	0.0%	5.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	23.5%	23.5%	2.9%	0.0%	0.0%	5.9%	0.0%	0.0%	0.0%	5.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	
9	State	0	1	2	e	4	S	9	7	80	6	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	
	nked Interpret.	5), (17)		, (1)		(, (9), (21), (25)		, (18)		other states 22	), (4), (6)-(8)	0)-(15), (19), (20)	2)-(24), (26)-(31)																					2

*Figure 14.* Results for the Boltzmann machine in the 5-atom micro-world, tested against the neutral premise. States are given in decimal format.

nked Interpret.	State	Expected %	Samples	Output %	Samples	Output %	Samples	Output %	Samples	Output %	Samples	Output %	Samples	Output %	
), (17)	0	10.1%	4687	7.8%	5109	8.5%	4875	8.1%	4901	8.2%	5293	8.8%	24865	8.3%	
	1	10.1%	5560	9.3%	5305	8.8%	5466	9.1%	5562	9.3%	4957	8.3%	26850	9.0%	
(1)	2	2.5%	1517	2.5%	1391	2.3%	1368	2.3%	1829	3.0%	1490	2.5%	7595	2.5%	
	e	0.6%	1066	1.8%	1087	1.8%	828	1.4%	1050	1.8%	1200	2.0%	5231	1.7%	
(9), (21), (25)	4	0.6%	1433	2.4%	1620	2.7%	1651	2.8%	1284	2.1%	1546	2.6%	7534	2.5%	
	S	5.1%	3399	5.7%	3093	5.2%	2979	5.0%	3431	5.7%	3091	5.2%	15993	5.3%	
(18)	9	0.6%	741	1.2%	579	1.0%	729	1.2%	789	1.3%	678	1.1%	3516	1.2%	
	7	0.6%	685	1.1%	706	1.2%	708	1.2%	632	1.1%	860	1.4%	3591	1.2%	
other states 22	80	0.6%	1434	2.4%	1687	2.8%	1758	2.9%	1690	2.8%	1735	2.9%	8304	2.8%	
(4), (6)-(8)	6	5.1%	2990	5.0%	2844	4.7%	2932	4.9%	3161	5.3%	2885	4.8%	14812	4.9%	
)-(15), (19), (20)	10	0.6%	813	1.4%	811	1.4%	804	1.3%	720	1.2%	723	1.2%	3871	1.3%	
)-(24), (26)-(31)	11	0.6%	772	1.3%	737	1.2%	726	1.2%	675	1.1%	834	1.4%	3744	1.2%	
	12	0.6%	803	1.3%	798	1.3%	567	%6.0	568	0.9%	854	1.4%	3590	1.2%	
	13	0.6%	1605	2.7%	1619	2.7%	1427	2.4%	1626	2.7%	1287	2.1%	7564	2.5%	
	14	0.6%	672	1.1%	517	%6.0	416	0.7%	503	0.8%	497	0.8%	2605	%6.0	
	15	0.6%	505	0.8%	674	1.1%	748	1.2%	729	1.2%	559	%6:0	3215	1.1%	
	16	20.3%	5289	8.8%	5656	9.4%	5268	8.8%	5586	9.3%	5933	9.9%	27732	9.2%	
	17	20.3%	6513	10.9%	6337	10.6%	7118	11.9%	6267	10.4%	6190	10.3%	32425	10.8%	
	18	2.5%	1785	3.0%	1856	3.1%	1670	2.8%	1750	2.9%	1676	2.8%	8737	2.9%	
	19	0.6%	1184	2.0%	1145	1.9%	1297	2.2%	1192	2.0%	1252	2.1%	6070	2.0%	
	20	0.6%	1378	2.3%	1854	3.1%	1528	2.5%	1659	2.8%	1515	2.5%	7934	2.6%	
	21	5.1%	2902	4.8%	3183	5.3%	3054	5.1%	2977	5.0%	3343	5.6%	15459	5.2%	
	22	0.6%	777	1.3%	937	1.6%	833	1.4%	986	1.6%	796	1.3%	4329	1.4%	
	23	0.6%	921	1.5%	699	1.1%	868	1.4%	735	1.2%	792	1.3%	3985	1.3%	
	24	0.6%	1894	3.2%	1661	2.8%	1819	3.0%	1872	3.1%	1667	2.8%	8913	3.0%	
	25	5.1%	3526	5.9%	3223	5.4%	3413	5.7%	3208	5.3%	3345	5.6%	16715	5.6%	
	26	0.6%	876	1.5%	646	1.1%	922	1.5%	657	1.1%	723	1.2%	3824	1.3%	
	27	0.6%	787	1.3%	719	1.2%	930	1.6%	756	1.3%	852	1.4%	4044	1.3%	
	28	0.6%	783	1.3%	585	1.0%	527	%6.0	603	1.0%	873	1.5%	3371	1.1%	
	29	0.6%	1542	2.6%	1703	2.8%	1462	2.4%	1606	2.7%	1681	2.8%	7994	2.7%	
	30	0.6%	479	0.8%	533	%6'0	523	%6.0	481	0.8%	384	0.6%	2400	0.8%	
	31	0.6%	682	1.1%	716	1.2%	786	1.3%	515	0.9%	489	0.8%	3188	1.1%	
	Accumul	ated error =	60000	59.1%	60000	58.8%	60000	57.8%	60000	58.0%	60000	58.4%	300000	58.1%	÷

*Figure 15.* Results for the Boltzmann machine in the 5-atom micro-world, tested against the neutral premise. States are given in decimal format.

Stdev																																		+ 1 75
Avg	Output %	7.7%	19.0%	3.1%	%0.0	0.1%	9.9%	0.1%	0.0%	0.0%	9.7%	0.1%	0.0%	0.0%	1.0%	0.0%	%0.0	7.7%	19.1%	3.2%	0.0%	0.0%	8.8%	0.2%	%0.0	0.0%	8.9%	0.1%	0.0%	0.0%	1.0%	0.0%	0.0%	<b>A8 6%</b>
	Samples	23011	57056	9251	105	164	29819	409	51	118	29024	444	99	2	3058	2	34	22988	57166	9536	79	125	26525	478	65	136	26720	415	43	0	3103	7	9	SODOOD F
	Output %	8.9%	18.8%	3.0%	0.0%	0.1%	9.4%	0.1%	0.0%	0.0%	9.1%	0.2%	0.0%	0.0%	0.9%	0.0%	0.0%	7.6%	19.0%	3.4%	0.0%	0.0%	9.0%	0.1%	0.0%	0.0%	9.0%	0.2%	0.0%	0.0%	1.0%	0.0%	0.0%	AE 002
	Samples	5368	11281	1771	18	43	5614	72	4	27	5438	106	16	Ļ	541	0	1	4581	11385	2065	15	27	5414	72	10	29	5402	102	6	0	588	0	0	60000
	Output %	7.5%	20.0%	2.4%	%0.0	0.0%	10.3%	0.1%	0.0%	0.0%	9.1%	0.1%	0.0%	0.0%	0.6%	0.0%	%0.0	8.3%	19.9%	2.9%	%0.0	0.0%	9.4%	0.1%	0.0%	%0.0	8.2%	0.1%	0.0%	0.0%	0.6%	0.0%	0.0%	70 8%
	Samples	4479	11976	1443	20	22	6205	69	10	20	5461	70	12	0	369	0	12	4991	11965	1759	24	23	5613	86	7	24	4899	85	8	0	346	1	1	60000
	Output %	6.9%	19.1%	3.4%	%0.0	%0.0	9.3%	0.2%	%0.0	0.0%	9.3%	0.2%	0.0%	0.0%	%6.0	0.0%	%0.0	7.6%	20.2%	3.1%	%0.0	%0.0	8.8%	0.2%	%0.0	%0.0	9.4%	0.1%	%0.0	%0.0	1.1%	0.0%	0.0%	702 00
	Samples	4169	11435	2057	24	21	5574	92	S	26	5591	108	11	0	512	0	6	4545	12131	1859	14	27	5298	104	22	26	5623	70	4	0	641	0	2	60000
	Output %	7.5%	18.8%	3.2%	%0.0	0.1%	10.6%	0.1%	0.0%	0.0%	10.2%	0.1%	0.0%	0.0%	1.5%	0.0%	0.0%	8.0%	17.6%	3.3%	%0.0	0.0%	8.5%	0.2%	%0.0	0.1%	8.6%	0.1%	0.0%	0.0%	1.5%	0.0%	0.0%	48 1%
	Samples	4488	11261	1893	26	39	6338	83	22	6	6129	78	12	1	903	0	2	4793	10568	1951	12	26	5090	112	11	30	5150	79	12	0	880	0	2	60000
	Output %	7.5%	18.5%	3.5%	%0.0	0.1%	10.1%	0.2%	0.0%	0.1%	10.7%	0.1%	0.0%	0.0%	1.2%	0.0%	%0.0	6.8%	18.5%	3.2%	%0.0	0.0%	8.5%	0.2%	%0.0	0.0%	9.4%	0.1%	0.0%	0.0%	1.1%	0.0%	0.0%	70 07
	Samples	4507	11103	2087	17	39	6088	93	10	36	6405	82	15	0	733	2	10	4078	11117	1902	14	22	5110	104	15	27	5646	79	10	0	648	0	1	60000
Set: 120-U	Expected %	12.5%	12.5%	4.2%	0.0%	0.0%	8.3%	0.0%	0.0%	0.0%	8.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	16.7%	16.7%	4.2%	0.0%	0.0%	8.3%	0.0%	0.0%	0.0%	8.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	- acta pote
Iraining	State	0	1	2	e	4	S	9	7	80	6	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	Accumula
	anked Interpret.	.6), (17)		(1), (1)		(), (9), (21), (25)		!), (18)		ll other states 22	(8), (4), (6)-(8)	(0)-(15), (19), (20)	2)-(24), (26)-(31)																					1

*Figure 16.* Results for the Boltzmann machine in the 5-atom micro-world, tested against the neutral premise. States are given in decimal format.

terpret.	State	Expected %	Samples	Output %	Samples	Output %	Samples	Output %	Samples	Output %	Samples	Output %	Samples	Output %
	0	10.6%	4680	7.8%	4722	7.9%	5603	9.3%	4825	8.0%	5246	8.7%	25076	8.4%
	1	10.6%	4970	8.3%	5375	9.0%	5599	9.3%	5373	9.0%	5835	9.7%	27152	9.1%
	2	3.5%	1347	2.2%	1298	2.2%	1571	2.6%	1665	2.8%	1572	2.6%	7453	2.5%
	e	0.7%	930	1.6%	1130	1.9%	876	1.5%	830	1.4%	1062	1.8%	4828	1.6%
21), (25)	4	0.7%	1874	3.1%	1549	2.6%	1755	2.9%	1712	2.9%	1870	3.1%	8760	2.9%
	S	7.0%	3770	6.3%	3128	5.2%	3825	6.4%	3509	5.8%	3778	6.3%	18010	6.0%
	9	0.7%	769	1.3%	793	1.3%	770	1.3%	738	1.2%	729	1.2%	3799	1.3%
	7	0.7%	841	1.4%	653	1.1%	683	1.1%	868	1.5%	699	1.1%	3744	1.2%
states 22	8	0.7%	1527	2.5%	2174	3.6%	1881	3.1%	1803	3.0%	2007	3.3%	9392	3.1%
6)-(8)	6	7.0%	3693	6.2%	3784	6.3%	3600	6.0%	3433	5.7%	3905	6.5%	18415	6.1%
, (19), (20)	10	0.7%	744	1.2%	703	1.2%	716	1.2%	868	1.4%	846	1.4%	3877	1.3%
, (26)-(31)	11	0.7%	724	1.2%	888	1.5%	592	1.0%	681	1.1%	816	1.4%	3701	1.2%
	12	0.7%	752	1.3%	689	1.1%	773	1.3%	764	1.3%	686	1.1%	3664	1.2%
	13	0.7%	1929	3.2%	1856	3.1%	1942	3.2%	1851	3.1%	1670	2.8%	9248	3.1%
	14	0.7%	330	0.6%	339	0.6%	335	0.6%	355	0.6%	390	0.7%	1749	0.6%
	15	0.7%	721	1.2%	620	1.0%	380	0.6%	640	1.1%	632	1.1%	2993	1.0%
	16	14.1%	4956	8.3%	4791	8.0%	4843	8.1%	4922	8.2%	5017	8.4%	24529	8.2%
	17	14.1%	5562	9.3%	5522	9.2%	4962	8.3%	5251	8.8%	4670	7.8%	25967	8.7%
	18	3.5%	1676	2.8%	1506	2.5%	1453	2.4%	1787	3.0%	1683	2.8%	8105	2.7%
	19	0.7%	1010	1.7%	1104	1.8%	993	1.7%	897	1.5%	006	1.5%	4904	1.6%
	20	0.7%	1875	3.1%	1807	3.0%	1704	2.8%	1960	3.3%	1690	2.8%	9036	3.0%
	21	7.0%	3398	5.7%	3256	5.4%	3659	6.1%	3723	6.2%	3288	5.5%	17324	5.8%
	22	0.7%	746	1.2%	673	1.1%	634	1.1%	719	1.2%	778	1.3%	3550	1.2%
	23	0.7%	850	1.4%	743	1.2%	752	1.3%	793	1.3%	716	1.2%	3854	1.3%
	24	0.7%	1616	2.7%	2010	3.4%	1861	3.1%	1705	2.8%	1405	2.3%	8597	2.9%
	25	7.0%	3295	5.5%	3635	6.1%	3282	5.5%	3476	5.8%	2827	4.7%	16515	5.5%
	26	0.7%	809	1.3%	795	1.3%	627	1.0%	713	1.2%	825	1.4%	3769	1.3%
	27	0.7%	763	1.3%	847	1.4%	703	1.2%	728	1.2%	613	1.0%	3654	1.2%
	28	0.7%	748	1.2%	673	1.1%	760	1.3%	720	1.2%	723	1.2%	3624	1.2%
	29	0.7%	2022	3.4%	1789	3.0%	1806	3.0%	1833	3.1%	2091	3.5%	9541	3.2%
	30	0.7%	438	0.7%	459	0.8%	357	0.6%	358	0.6%	392	0.7%	2004	0.7%
	31	0.7%	635	1.1%	689	1.1%	703	1.2%	470	0.8%	699	1.1%	3166	1.1%
	Accumul	ated error =	60000	56.1%	60000	56.7%	60000	53.8%	60000	55.3%	60000	54.1%	300000	55.2%

*Figure 17.* Results for the Boltzmann machine in the 5-atom micro-world, tested against the neutral premise. States are given in decimal format.



*Figure 18.* Results for the Boltzmann machine in the 5-atom micro-world, tested against specific premises. States are given in decimal format.



*Figure 19.* Results for the Boltzmann machine in the 5-atom micro-world, tested against specific premises. States are given in decimal format.



*Figure 20.* Results for the Boltzmann machine in the 5-atom micro-world, tested against specific premises. States are given in decimal format.

	Training	Set: 180-0												Avg
Ranked Interpret.	State	Expected %	Samples	Output %										
(32), (33)	0	11.1%	6244	10.4%	6065	10.1%	6133	10.2%	7025	11.7%	6822	11.4%	32289	10.8%
	1	11.1%	7667	12.8%	8139	13.6%	8446	14.1%	8307	13.8%	7658	12.8%	40217	13.4%
(0), (1)	2	2.8%	3030	5.1%	2677	4.5%	2698	4.5%	2996	5.0%	3347	5.6%	14748	4.9%
	3	0.0%	31	0.1%	28	0.0%	29	0.0%	21	0.0%	19	0.0%	128	0.0%
(9), (17), (41), (49)	4	5.6%	2534	4.2%	2430	4.1%	2562	4.3%	2270	3.8%	2490	4.2%	12286	4.1%
	5	0.0%	25	0.0%	24	0.0%	31	0.1%	41	0.1%	31	0.1%	152	0.1%
(4), (36)	6	0.0%	216	0.4%	171	0.3%	217	0.4%	228	0.4%	249	0.4%	1081	0.4%
	7	0.0%	0	0.0%	0	0.0%	1	0.0%	2	0.0%	2	0.0%	5	0.0%
(2), (34)	8	0.0%	35	0.1%	33	0.1%	34	0.1%	31	0.1%	29	0.0%	162	0.1%
	9	8.3%	5054	8.4%	5447	9.1%	4821	8.0%	4716	7.9%	4928	8.2%	24966	8.3%
All other states 52	10	0.0%	92	0.2%	59	0.1%	55	0.1%	33	0.1%	67	0.1%	306	0.1%
(3), (5)-(8), (10)-(16),	11	0.0%	34	0.1%	49	0.1%	24	0.0%	23	0.0%	25	0.0%	155	0.1%
(18)-(31),(35), (37)-(40)	12	0.0%	38	0.1%	20	0.0%	35	0.1%	25	0.0%	25	0.0%	143	0.0%
(42)-(48), (50)-(63)	13	0.0%	52	0.1%	61	0.1%	67	0.1%	89	0.1%	64	0.1%	333	0.1%
	14	0.0%	19	0.0%	16	0.0%	22	0.0%	42	0.1%	13	0.0%	112	0.0%
	15	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%
	16	0.0%	22	0.0%	22	0.0%	37	0.1%	33	0.1%	30	0.1%	144	0.0%
	17	8.3%	5033	8.4%	5051	8.4%	4425	7.4%	5086	8.5%	4857	8.1%	24452	8.2%
	18	0.0%	21	0.0%	39	0.1%	29	0.0%	43	0.1%	36	0.1%	168	0.1%
	19	0.0%	31	0.1%	23	0.0%	39	0.1%	40	0.1%	26	0.0%	159	0.1%
	20	0.0%	31	0.1%	22	0.0%	20	0.0%	27	0.0%	19	0.0%	119	0.0%
	21	0.0%	32	0.1%	65	0.1%	48	0.1%	68	0.1%	77	0.1%	290	0.1%
	22	0.0%	51	0.1%	53	0.1%	37	0.1%	32	0.1%	22	0.0%	195	0.1%
	23	0.0%	0	0.0%	1	0.0%	0	0.0%	0	0.0%	0	0.0%	1	0.0%
	24	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%
	25	0.0%	358	0.6%	180	0.3%	495	0.8%	320	0.5%	333	0.6%	1686	0.6%
	26	0.0%	3	0.0%	0	0.0%	0	0.0%	1	0.0%	1	0.0%	5	0.0%
	27	0.0%	5	0.0%	13	0.0%	9	0.0%	4	0.0%	14	0.0%	45	0.0%
	28	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%
	29	0.0%	23	0.0%	16	0.0%	24	0.0%	27	0.0%	19	0.0%	109	0.0%
	30	0.0%	5	0.0%	0	0.0%	1	0.0%	0	0.0%	0	0.0%	6	0.0%
	31	0.0%	3	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	3	0.0%
	32	13.9%	6144	10.2%	6329	10.5%	6576	11.0%	6245	10.4%	6615	11.0%	31909	10.6%
	33	13.9%	7846	13.1%	7/58	12.9%	8098	13.5%	6909	11.5%	7329	12.2%	37940	12.6%
	39	2.8%	2000	4.475	2891	9.8%	2027	9.9%	29/1	5.0%	2609	9.3%	13764	4.075
	35	0.0%	2120	3.6%	2842	4 7%	23	3.8%	18	0.0%	2450	4.1%	12101	0.0%
	30	0.0%	10	0.0%	2043	9.770	2291	0.1%	24/9	4.170	2450	9.179	141	9.179
	37	0.0%	190	0.0%	222	0.1%	106	0.1%	100	0.1%	20	0.0%	1092	0.0%
	39	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%
	40	0.0%	25	0.0%	24	0.0%	29	0.0%	18	0.0%	29	0.0%	125	0.0%
	41	8.3%	4491	7.5%	4316	7.2%	4314	7.2%	3921	6.5%	4429	7.4%	21471	7.2%
	42	0.0%	41	0.1%	49	0.1%	39	0.1%	49	0.1%	30	0.1%	208	0.1%
	43	0.0%	19	0.0%	22	0.0%	21	0.0%	26	0.0%	23	0.0%	111	0.0%
	44	0.0%	27	0.0%	39	0.1%	16	0.0%	20	0.0%	16	0.0%	118	0.0%
	45	0.0%	52	0.1%	35	0.1%	61	0.1%	72	0.1%	35	0.1%	255	0.1%
	46	0.0%	10	0.0%	53	0.1%	20	0.0%	26	0.0%	27	0.0%	136	0.0%
	47	0.0%	0	0.0%	0	0.0%	0	0.0%	1	0.0%	0	0.0%	1	0.0%
	48	0.0%	32	0.1%	22	0.0%	24	0.0%	31	0.1%	32	0.1%	141	0.0%
	49	8.3%	4773	8.0%	4203	7.0%	4516	7.5%	4877	8.1%	4475	7.5%	22844	7.6%
	50	0.0%	30	0.1%	25	0.0%	23	0.0%	41	0.1%	15	0.0%	134	0.0%
	51	0.0%	28	0.0%	24	0.0%	16	0.0%	41	0.1%	19	0.0%	128	0.0%
	52	0.0%	35	0.1%	17	0.0%	43	0.1%	33	0.1%	24	0.0%	152	0.1%
	53	0.0%	72	0.1%	52	0.1%	65	0.1%	71	0.1%	46	0.1%	306	0.1%
	54	0.0%	70	0.1%	43	0.1%	49	0.1%	54	0.1%	23	0.0%	239	0.1%
	55	0.0%	0	0.0%	1	0.0%	0	0.0%	0	0.0%	0	0.0%	1	0.0%
	56	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%
	57	0.0%	511	0.9%	193	0.3%	527	0.9%	292	0.5%	241	0.4%	1764	0.6%
	58	0.0%	0	0.0%	2	0.0%	0	0.0%	0	0.0%	0	0.0%	2	0.0%
	59	0.0%	47	0.1%	31	0.1%	7	0.0%	26	0.0%	12	0.0%	123	0.0%
	60	0.0%	0	0.0%	0	0.0%	0	0.0%	1	0.0%	0	0.0%	1	0.0%
	61	0.0%	56	0.1%	38	0.1%	42	0.1%	19	0.0%	4	0.0%	159	0.1%
	62	0.0%	0	0.0%	0	0.0%	0	0.0%	2	0.0%	0	0.0%	2	0.0%
	63	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%	0	0.0%
	60	0.070			-		-				-			

*Figure 21.* Results for the Boltzmann machine in the 6-atom micro-world, tested against the neutral premise. States are given in decimal format.

	Training	Set: 232-1														Avg
Ranked Interpret.	State	Expected %	ſ	Samples	Output %	[	Samples	Output %								
(32), (33)	0	8.6%	ľ	2900	4.8%	3311	5.5%	3289	5.5%	2937	4.9%	3264	5.4%	1	15701	5.2%
	1	8.6%		3252	5.4%	4225	7.0%	3162	5.3%	3357	5.6%	3846	6.4%		17842	5.9%
(0), (1)	2	2.2%		1460	2.4%	1364	2.3%	1372	2.3%	1439	2.4%	1221	2.0%		6856	2.3%
	3	0.4%		919	1.5%	852	1.4%	920	1.5%	1063	1.8%	1111	1.9%		4865	1.6%
(9), (17), (41), (49)	4	4.3%		1891	3.2%	1780	3.0%	1920	3.2%	1789	3.0%	1942	3.2%		9322	3.1%
	5	0.4%		1201	2.0%	1383	2.3%	1344	2.2%	1050	1.8%	1225	2.0%		6203	2.1%
(4), (36)	6	0.4%		665	1.1%	731	1.2%	586	1.0%	717	1.2%	737	1.2%		3436	1.1%
	7	0.4%		478	0.8%	419	0.7%	520	0.9%	364	0.6%	430	0.7%		2211	0.7%
(2), (34)	8	0.4%		877	1.5%	870	1.5%	906	1.5%	839	1.4%	1109	1.8%		4601	1.5%
	9	6.5%		2129	3.5%	2200	3.7%	2056	3.4%	2332	3.9%	2271	3.8%		10988	3.7%
All other states 52	10	0.4%		492	0.8%	524	0.9%	444	0.7%	557	0.9%	518	0.9%		2535	0.8%
(3), (5)-(8), (10)-(16),	11	0.4%		577	1.0%	661	1.1%	573	1.0%	780	1.3%	601	1.0%		3192	1.1%
(18)-(31),(35), (37)-(40)	12	0.4%	- 1	533	0.9%	615	1.0%	632	1.1%	628	1.0%	567	0.9%		2975	1.0%
(42)-(48), (50)-(63)	13	0.4%		789	1.3%	769	1.3%	889	1.5%	742	1.2%	801	1.3%		3990	1.3%
	14	0.4%		590	1.0%	524	0.9%	577	1.0%	609	1.0%	565	0.9%		2865	1.0%
	15	0.4%		341	0.6%	339	0.6%	322	0.5%	364	0.6%	235	0.4%		1601	0.5%
	16	0.4%	- 1	1180	2.0%	1193	2.0%	1089	1.8%	1064	1.8%	1006	1.7%		5532	1.8%
	17	6.5%		2518	4.2%	2229	3.7%	2403	4.0%	2131	3.6%	2103	3.5%		11384	3.8%
	18	0.4%	- 1	535	0.9%	589	1.0%	567	0.9%	520	0.9%	490	0.8%	_	2701	0.9%
	19	0.4%		574	1.0%	605	1.0%	523	0.9%	704	1.2%	628	1.0%		3034	1.0%
	20	0.4%	- 1	661	1.1%	577	1.0%	860	1.4%	586	1.0%	586	1.0%	_	3270	1.1%
	21	0.4%	- 1	987	1.6%	822	1.4%	795	1.3%	965	1.6%	738	1.2%	_	4307	1.4%
	22	0.4%	- 1	550	0.9%	548	0.9%	502	0.8%	303	0.5%	506	0.8%	_	2409	0.8%
	23	0.4%		298	0.5%	448	0.7%	332	0.6%	286	0.5%	267	0.4%		1631	0.5%
	24	0.4%		414	0.7%	424	0.7%	363	0.6%	332	0.6%	431	0.7%		1964	0.7%
	25	0.4%		969	1.6%	830	1.4%	924	1.5%	990	1.7%	896	1.5%		4609	1.5%
	26	0.4%	- 1	248	0.4%	242	0.4%	166	0.3%	424	0.7%	311	0.5%	_	1391	0.5%
	27	0.4%		553	0.9%	653	1.1%	533	0.9%	632	1.1%	531	0.9%		2902	1.0%
	28	0.4%	- 1	411	0.7%	308	0.5%	386	0.6%	256	0.4%	453	0.8%	_	1814	0.6%
	29	0.4%		468	0.8%	567	0.9%	620	1.0%	527	0.9%	537	0.9%		2719	0.9%
	30	0.4%		239	0.4%	234	0.4%	318	0.5%	320	0.5%	283	0.5%		1394	0.5%
	31	0.4%		256	0.4%	625	1.0%	425	0.7%	369	0.6%	284	0.5%		1959	0.7%
	32	10.8%		3072	5.1%	3182	5.3%	3289	5.5%	3367	5.6%	3458	5.8%		16368	5.5%
	33	10.8%	- 1	3294	5.5%	3101	5.2%	3493	5.8%	3558	5.9%	3966	6.6%	_	17412	5.8%
	34	2.2%	- 1	1557	2.6%	1346	2.2%	1053	1.8%	1475	2.5%	1342	2.2%	_	6773	2.3%
	35	0.4%	- 1	972	1.6%	919	1.5%	946	1.6%	1071	1.8%	845	1.4%	_	4753	1.6%
	36	4.3%	- 1	1954	3.3%	1535	2.6%	1827	3.0%	1710	2.9%	1600	2.7%	_	8626	2.9%
	37	0.4%	- 1	1129	1.9%	1101	1.8%	1071	1.8%	1041	1.7%	969	1.6%	_	5311	1.8%
	38	0.4%	- 1	627	1.0%	506	0.8%	477	0.8%	677	1.1%	665	1.1%	_	2952	1.0%
	39	0.4%	- 1	436	0.7%	369	0.6%	371	0.6%	330	0.6%	483	0.8%	_	1989	0.7%
	40	0.4%		978	1.6%	769	1.3%	935	1.6%	979	1.6%	884	1.5%		4545	1.5%
	41	6.5%		1958	3.3%	1955	3.3%	2254	3.8%	2139	3.6%	2139	3.6%		10445	3.5%
	42	0.4%		414	0.7%	503	0.8%	553	0.9%	552	0.9%	517	0.9%		2539	0.8%
	43	0.4%		519	0.9%	622	1.0%	451	0.8%	617	1.0%	574	1.0%		2783	0.9%
	44	0.4%		759	1.3%	635	1.1%	583	1.0%	723	1.2%	573	1.0%		3273	1.1%
	45	0.4%		832	1.4%	619	1.0%	800	1.3%	695	1.2%	598	1.0%		3544	1.2%
	46	0.4%		516	0.9%	549	0.9%	481	0.8%	624	1.0%	410	0.7%		2580	0.9%
	47	0.4%	- 1	411	0.7%	262	0.4%	436	0.7%	395	0.7%	329	0.5%	_	1833	0.6%
	48	0.4%	- 1	1030	1.7%	1025	1.7%	1192	2.0%	1020	1.7%	1230	2.1%	_	5497	1.8%
	49	6.5%	- 1	2212	3.7%	2278	3.8%	2305	3.8%	1976	3.3%	2315	3.9%	_	11086	3.7%
	50	0.4%	- 1	603	1.0%	631	1.1%	563	0.9%	557	0.9%	526	0.9%	_	2880	1.0%
	51	0.4%		633	1.1%	611	1.0%	734	1.2%	702	1.2%	577	1.0%		3257	1.1%
	52	0.4%		722	1.2%	519	0.9%	579	1.0%	688	1.1%	466	0.8%		2974	1.0%
	53	0.4%		847	1.4%	867	1.4%	686	1.1%	773	1.3%	579	1.0%		3752	1.3%
	54	0.4%	- 1	676	1.1%	550	0.9%	354	0.6%	390	0.7%	610	1.0%	_	2580	0.9%
	55	0.4%	- 1	333	0.6%	336	0.6%	213	0.4%	191	0.3%	318	0.5%	_	1391	0.5%
	56	0.4%	- 1	401	0.7%	350	0.6%	479	0.8%	509	0.8%	422	0.7%	_	2161	0.7%
	57	0.4%		886	1.5%	901	1.5%	1126	1.9%	860	1.4%	926	1.5%		4699	1.6%
	58	0.4%		252	0.4%	307	0.5%	233	0.4%	240	0.4%	344	0.6%		1376	0.5%
	59	0.4%		478	0.8%	428	0.7%	602	1.0%	608	1.0%	522	0.9%		2638	0.9%
	60	0.4%		391	0.7%	419	0.7%	398	0.7%	410	0.7%	239	0.4%		1857	0.6%
	61	0.4%		687	1.1%	621	1.0%	477	0.8%	649	1.1%	481	0.8%		2915	1.0%
	62	0.4%	- 1	224	0.4%	301	0.5%	371	0.6%	293	0.5%	275	0.5%	_	1464	0.5%
,	63	0.4%	L	242	0.4%	422	0.7%	350	0.6%	205	0.3%	325	0.5%	. l	1544	0.5%

*Figure 22.* Results for the Boltzmann machine in the 6-atom micro-world, tested against the neutral premise. States are given in decimal format.



*Figure 23.* Results for the Boltzmann machine in the 6-atom micro-world, tested against specific premises. States are given in decimal format.



*Figure 24.* Results for the Boltzmann machine in the 6-atom micro-world, tested against specific premises. States are given in decimal format.

Stdev																		± 1.18	± 0.07																			± 2.01
Avg	Output %	0.3%	%6.0	29.6%	0.5%	0.5%	19.1%	0.5%	0.5%	0.3%	0.8%	32.9%	0.5%	0.5%	12.5%	0.5%	0.5%	140.7%	8.8%		Output %	0.0%	%0.0	4.0%	5.0%	2.6%	0.3%	17.2%	15.0%	%0.0	%0.0	8.0%	7.9%	2.9%	0.2%	18.8%	18.1%	166.4%
	Activations	0.0019	0.0058	0.1995	0.0031	0.0031	0.1288	0.0031	0.0031	0.0021	0.0052	0.2215	0.0031	0.0031	0.0842	0.0031	0.0031	0.6738			Activations	0.0000	0.0000	0.0277	0.0344	0.0176	0.0021	0.1182	0.1033	0.0000	0.0000	0.0551	0.0541	0.0196	0.0012	0.1292	0.1239	0.6864
	Output %	0.2%	%6.0	31.8%	0.4%	0.4%	21.7%	0.4%	0.4%	0.5%	0.6%	28.6%	0.4%	0.4%	12.5%	0.4%	0.4%	140.8%	8.8%		Output %	0.0%	%0.0	11.5%	1.6%	13.0%	0.0%	20.3%	6.6%	%0.0	%0.0	0.8%	17.0%	0.1%	0.2%	13.0%	15.8%	175.2%
	Activations	0.0003	0.0013	0.0468	0.0006	0.0006	0.0319	0.0006	0.0006	0.0007	0.0009	0.0420	0.0006	0.0006	0.0184	0.0006	0.0006	0.1471			Activations	0.0000	0.0000	0.0123	0.0017	0.0140	0.0000	0.0218	0.0071	0.0000	0.0000	0.0009	0.0182	0.0001	0.0002	0.0140	0.0170	0.1073
	Output %	0.6%	0.8%	29.5%	0.6%	0.6%	17.4%	0.6%	0.6%	0.4%	1.0%	30.9%	0.6%	0.6%	14.4%	0.6%	0.6%	139.5%	8.7%		Output %	0.0%	%0.0	0.1%	3.7%	0.1%	0.0%	4.3%	30.3%	%0.0	%0.0	11.6%	4.6%	%0.0	%0.0	13.7%	31.6%	179.0%
	Activations	0.0006	0.0008	0.0289	0.0006	0.0006	0.0170	0.0006	0.0006	0.0004	0.0010	0.0303	0.0006	0.0006	0.0141	0.0006	0.0006	0.0979			Activations	0.0000	0.0000	0.0001	0.0050	0.0002	0.0000	0.0058	0.0410	0.0000	0.0000	0.0157	0.0063	0.0000	0.0000	0.0186	0.0428	0.1355
	Output %	0.3%	1.3%	27.5%	0.5%	0.5%	18.5%	0.5%	0.5%	0.3%	1.0%	36.6%	0.5%	0.5%	10.1%	0.5%	0.5%	139.3%	8.7%		Output %	0.0%	%0.0	0.7%	0.4%	0.0%	1.6%	20.4%	14.3%	0.0%	%0.0	8.1%	5.0%	6.8%	0.2%	30.8%	11.7%	173.6%
	Activations	0.0004	0.0017	0.0355	0.0007	0.0007	0.0238	0.0007	0.0007	0.0004	0.0013	0.0472	0.0007	0.0007	0.0130	0.0007	0.0007	0.1289			Activations	0.0000	0.0000	0.0008	0.0005	0.0000	0.0019	0.0241	0.0169	0.0000	0.0000	0.0096	0.0059	0.0080	0.0002	0.0363	0.0138	0.1180
	Output %	0.3%	0.3%	28.7%	0.4%	0.4%	17.8%	0.4%	0.4%	0.2%	0.8%	33.5%	0.4%	0.4%	15.0%	0.4%	0.4%	141.9%	8.9%		Output %	0.0%	0.0%	10.2%	13.3%	%0.0	0.1%	22.5%	10.8%	%0.0	%0.0	1.4%	13.6%	0.1%	0.6%	14.1%	13.2%	175.0%
	Activations	0.0004	0.0005	0.0412	0.0006	0.0006	0.0256	0.0006	0.0006	0.0003	0.0012	0.0482	0.0006	0.0006	0.0215	0.0006	0.0006	0.1437			Activations	0.0000	0.0000	0.0140	0.0182	0.0000	0.0002	0.0308	0.0147	0.0000	0.0000	0.0019	0.0186	0.0001	0.0008	0.0193	0.0181	0.1367
	Output %	0.1%	1.0%	30.2%	0.4%	0.4%	19.5%	0.4%	0.4%	0.2%	0.5%	34.4%	0.4%	0.4%	11.0%	0.4%	0.4%	141.6%	8.9%		Output %	0.0%	0.0%	0.3%	4.8%	1.8%	%0.0	18.9%	12.5%	%0.0	0.0%	14.3%	2.7%	6.0%	%0.0	21.7%	17.0%	174.9%
	Activations	0.0002	0.0015	0.0471	0.0006	0.0006	0.0305	0.0006	0.0006	0.0003	0.0008	0.0538	0.0006	0.0006	0.0172	0.0006	0.0006	0.1562			Activations	0.0000	0.0000	0.0005	0600.0	0.0034	0.0000	0.0357	0.0236	0.0000	0.0000	0.0270	0.0051	0.0114	0.0000	0.0410	0.0322	0.1889
Set: 88-0	Expected %	18.1%	18.1%	4.5%	%0.0	0.0%	9.1%	0.0%	0.0%	18.1%	18.1%	4.5%	%0.0	%0.0	9.1%	%0.0	0.0%	ated error =	<pre>per state =</pre>	Set: 96-1	Expected %	16.7%	16.7%	4.2%	1.0%	1.0%	8.3%	1.0%	1.0%	16.7%	16.7%	4.2%	1.0%	1.0%	8.3%	1.0%	1.0%	ated error =
Training	State	0	1	2	ŝ	4	5	9	7	80	6	10	11	12	13	14	15	Accumula	Avg. error	Training	State	0	1	2	ß	4	5	9	7	80	6	10	11	12	13	14	15	Accumula
	anked Interpret.	0), (1), (8), (9)		5), (13)		2), (10)		3), (4), (6), (7),	11), (12), (14), (15)												anked Interpret.	0), (1), (8), (9)		5), (13)		2), (10)		3), (4), (6), (7),	11), (12), (14), (15)									

Appendix C: Multi-Layer Perceptron (MLP) Results

*Figure 25.* Results for the MLP in the 4-atom micro-world, tested against the neutral premise. States are given in decimal format.

anked Interpret.	State	Expected %	Activations	Output %	Activations	Output %	Activations	Output %	Activations	Output %	Activations	Output %	Activations	Output %	
0), (1), (8), (9)	0	16.7%	0.0003	0.3%	0.0003	0.2%	0.0001	0.1%	0.0001	0.1%	0.0005	0.3%	0.0013	0.2%	
	1	16.7%	0.0008	0.7%	0.0012	0.8%	0.0002	0.1%	0.0005	0.3%	0.0011	0.8%	0.0038	0.5%	
5), (13)	2	5.6%	0.0385	33.5%	0.0479	32.0%	0.0554	31.2%	0.0823	51.2%	0.0655	45.0%	0.2896	38.7%	
	m	0.0%	0.0005	0.4%	0.0005	0.3%	0.0005	0.3%	0.0005	0.3%	0.0005	0.3%	0.0025	0.3%	
(10) (10)	4	0.0%	0.0005	0.4%	0.0005	0.3%	0.0005	0.3%	0.0005	0.3%	0.0005	0.3%	0.0025	0.3%	
	2	11.1%	0.0273	23.8%	0.0168	11.2%	0.0450	25.4%	0.0176	11.0%	0.0246	16.9%	0.1313	17.6%	
(), (4), (6), (7),	9	0.0%	0.0005	0.4%	0.0005	0.3%	0.0005	0.3%	0.0005	0.3%	0.0005	0.3%	0.0025	0.3%	
1), (12), (14), (15)	7	0.0%	0.0005	0.4%	0.0005	0.3%	0.0005	0.3%	0.0005	0.3%	0.0005	0.3%	0.0025	0.3%	
	80	16.7%	0.0003	0.3%	0.0002	0.1%	0.0001	0.1%	0.0002	0.1%	0.0002	0.1%	0.0010	0.1%	
	6	16.7%	0.0005	0.4%	0.0008	0.5%	0.0003	0.2%	0.0002	0.1%	0.0007	0.5%	0.0025	0.3%	
	10	5.6%	0.0306	26.7%	0.0541	36.2%	0.0361	20.3%	0.0280	17.4%	0.0335	23.0%	0.1823	24.4%	
	11	0.0%	0.0005	0.4%	0.0005	0.3%	0.0005	0.3%	0.0005	0.3%	0.0006	0.4%	0.0026	0.3%	
	12	0.0%	0.0005	0.4%	0.0005	0.3%	0.0005	0.3%	0.0005	0.3%	0.0005	0.3%	0.0025	0.3%	
	13	11.1%	0.0125	10.9%	0.0242	16.2%	0.0362	20.4%	0.0278	17.3%	0.0154	10.6%	0.1161	15.5%	
	14	0.0%	0.0005	0.4%	0.0005	0.3%	0.0005	0.3%	0.0005	0.3%	0.0005	0.3%	0.0025	0.3%	
	15	0.0%	0.0005	0.4%	0.0005	0.3%	0.0005	0.3%	0.0005	0.3%	0.0005	0.3%	0.0025	0.3%	
2	Accumu	ated error =	0.1148	130.5%	0.1495	130.1%	0.1774	132.6%	0.1607	132.5%	0.1456	131.0%	0.7480	131.1%	± 1.15
	Avg. erro	or per state =		8.2%		8.1%		8.3%		8.3%		8.2%		8.2%	± 0.07
	Training	Set: 98-1													
anked Interpret.	State	Expected %	Activations	Output %	Activations	Output %	Activations	Output %	Activations	Output %	Activations	Output %	Activations	Output %	
(1), (1), (8), (9)	0	15.3%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	H	15.3%	0.0000	0.0%	0.0001	0.1%	0.0000	0.0%	0.0000	%0.0	0.0000	0.0%	0.0001	0.0%	
(13) (13)	2	5.1%	0.0012	%6.0	0.0256	22.2%	0.0138	8.4%	0.0001	0.1%	0.0007	0.7%	0.0414	6.6%	
	ß	1.0%	0.0044	3.2%	0.0044	3.8%	0.0164	10.0%	0.0051	4.4%	0.0020	2.0%	0.0323	5.1%	
2), (10)	4	1.0%	0.0001	0.1%	0.0000	0.0%	0.0007	0.4%	0.0011	1.0%	0.0000	%0.0	0.0019	0.3%	
	S	10.2%	0.0000	%0.0	0.0000	%0.0	0.0001	0.1%	0.0000	%0.0	0.0002	0.2%	0.0003	0.0%	
3), (4), (6), (7),	9	1.0%	0.0291	21.4%	0.0129	11.2%	0.0303	18.5%	0.0352	30.6%	0.0262	25.9%	0.1337	21.2%	
(1), (12), (14), (15)	7	1.0%	0.0518	38.1%	0.0321	27.9%	0.0303	18.5%	0.0102	8.9%	0.0242	23.9%	0.1486	23.6%	
	80	15.3%	0.0000	%0.0	0.0000	%0.0	0.0000	%0.0	0.0000	%0.0	0.0000	%0.0	0.0000	%0.0	
	6	15.3%	0.0000	%0.0	0.0000	0.0%	0.0000	%0.0	0.0000	%0.0	0.0000	0.0%	0.0000	0.0%	
	10	5.1%	0.0007	0.5%	0.0117	10.2%	0.0197	12.1%	0.0063	5.5%	0.0108	10.7%	0.0492	7.8%	
	11	1.0%	0.0214	15.7%	0.0081	7.0%	0.0054	3.3%	0.0017	1.5%	0.0013	1.3%	0.0379	6.0%	
	12	1.0%	0.0000	%0.0	0.0010	%6.0	0.0001	0.1%	0.0001	0.1%	0.0004	0.4%	0.0016	0.3%	
	13	10.2%	0.0018	1.3%	0.0000	%0.0	0.0000	%0.0	0.0000	%0.0	0.0001	0.1%	0.0019	0.3%	
	14	1.0%	0.0254	18.7%	0.0018	1.6%	0.0139	8.5%	0.0307	26.7%	0.0063	6.2%	0.0781	12.4%	
	15	1.0%	0.0002	0.2%	0.0174	15.1%	0.0327	20.0%	0.0244	21.2%	0.0291	28.7%	0.1038	16.5%	
2	Accumu	ated error =	0.1361	183.9%	0.1151	165.5%	0.1634	166.3%	0.1149	175.3%	0.1013	174.8%	0.6308	165.6%	± 7.56

*Figure 26.* Results for the MLP in the 4-atom micro-world, tested against the neutral premise. States are given in decimal format.



*Figure 27.* Results for the MLP in the 4-atom micro-world, tested against specific premises. States are given in decimal format.



*Figure 28.* Results for the MLP in the 4-atom micro-world, tested against specific premises. States are given in decimal format.



*Figure 29.* Results for the MLP in the 4-atom micro-world, tested against specific premises. States are given in decimal format.



*Figure 30.* Results for the MLP in the 4-atom micro-world, tested against specific premises. States are given in decimal format.

Output %	17.6%	0.1%	8.0%	1.6%	1.6%	12.7%	1.6%	1.6%	1.6%	13.1%	1.6%	1.6%	1.6%	1.6%	1.6%	1.6%	2.3%	0.0%	3.3%	1.6%	1.6%	3.5%	1.6%	1.6%	1.6%	4.0%	1.6%	1.6%	1.6%	1.6%	1.6%	1.6%
Activations	0.0669	0.0003	0.0303	0.0061	0.0062	0.0485	0.0061	0.0062	0.0062	0.0501	0.0062	0.0061	0.0061	0.0062	0.0061	0.0061	0.0089	0.0000	0.0126	0.0061	0.0062	0.0134	0.0061	0.0061	0.0061	0.0153	0.0061	0.0061	0.0061	0.0061	0.0061	0.0061
Output %	19.1%	0.1%	9.3%	1.7%	1.7%	10.1%	1.7%	1.7%	1.7%	11.6%	1.7%	1.7%	1.7%	1.7%	1.7%	1.7%	2.6%	0.0%	2.7%	1.7%	1.7%	3.1%	1.7%	1.7%	1.7%	4.6%	1.7%	1.7%	1.7%	1.7%	1.7%	1.7%
Activations	0.0148	0.0001	0.0072	0.0013	0.0013	0.0078	0.0013	0.0013	0.0013	0600.0	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0020	0.0000	0.0021	0.0013	0.0013	0.0024	0.0013	0.0013	0.0013	0.0036	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013
Output %	25.6%	0.1%	7.5%	1.4%	1.4%	12.2%	1.4%	1.4%	1.4%	12.1%	1.4%	1.4%	1.4%	1.4%	1.4%	1.4%	1.9%	0.0%	3.6%	1.4%	1.4%	2.9%	1.4%	1.4%	1.4%	3.0%	1.4%	1.4%	1.4%	1.4%	1.4%	1.4%
Activations	0.0237	0.0001	0.0069	0.0013	0.0013	0.0113	0.0013	0.0013	0.0013	0.0112	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0018	0.0000	0.0033	0.0013	0.0013	0.0027	0.0013	0.0013	0.0013	0.0028	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013
Output %	14.7%	0.0%	8.0%	1.5%	1.7%	11.9%	1.5%	1.7%	1.7%	17.6%	1.7%	1.5%	1.5%	1.7%	1.5%	1.5%	2.1%	0.0%	3.7%	1.5%	1.7%	3.4%	1.5%	1.5%	1.5%	3.8%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%
Activations	0.0105	0.0000	0.0057	0.0011	0.0012	0.0085	0.0011	0.0012	0.0012	0.0125	0.0012	0.0011	0.0011	0.0012	0.0011	0.0011	0.0015	0.0000	0.0026	0.0011	0.0012	0.0024	0.0011	0.0011	0.0011	0.0027	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011
Output %	13.6%	0.0%	7.7%	1.7%	1.7%	14.0%	1.7%	1.7%	1.7%	13.6%	1.7%	1.7%	1.7%	1.7%	1.7%	1.7%	2.2%	0.0%	3.4%	1.7%	1.7%	4.1%	1.7%	1.7%	1.7%	4.5%	1.7%	1.7%	1.7%	1.7%	1.7%	1.7%
Activations	0.0097	0.0000	0.0055	0.0012	0.0012	0.0100	0.0012	0.0012	0.0012	0.0097	0.0012	0.0012	0.0012	0.0012	0.0012	0.0012	0.0016	0.0000	0.0024	0.0012	0.0012	0.0029	0.0012	0.0012	0.0012	0.0032	0.0012	0.0012	0.0012	0.0012	0.0012	0.0012
Output %	12.0%	0.1%	7.3%	1.8%	1.8%	15.9%	1.8%	1.8%	1.8%	11.2%	1.8%	1.8%	1.8%	1.8%	1.8%	1.8%	2.9%	0.0%	3.2%	1.8%	1.8%	4.4%	1.8%	1.8%	1.8%	4.4%	1.8%	1.8%	1.8%	1.8%	1.8%	1.8%
Activations	0.0082	0.0001	0.0050	0.0012	0.0012	0.0109	0.0012	0.0012	0.0012	0.0077	0.0012	0.0012	0.0012	0.0012	0.0012	0.0012	0.0020	0.0000	0.0022	0.0012	0.0012	0.0030	0.0012	0.0012	0.0012	0.0030	0.0012	0.0012	0.0012	0.0012	0.0012	0.0012
Expected %	11.8%	11.8%	2.9%	0.0%	0.0%	5.9%	0.0%	0.0%	0.0%	5.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	23.5%	23.5%	2.9%	0.0%	0.0%	5.9%	0.0%	0.0%	0.0%	5.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
State	0	1	2	e	4	5	9	7	80	6	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
anked Interpret.	16), (17)		0), (1)		5), (9), (21), (25)		2), (18)		Il other states 22	3), (4), (6)-(8)	10)-(15), (19), (20)	22)-(24), (26)-(31)																				

*Figure 31.* Results for the MLP in the 5-atom micro-world, tested against the neutral premise. States are given in decimal format.

																																	± 0.55	
Output %	8.3%	9.0%	2.5%	1.7%	2.5%	5.3%	1.2%	1.2%	2.8%	4.9%	1.3%	1.2%	1.2%	2.5%	0.9%	1.1%	9.2%	10.8%	2.9%	2.0%	2.6%	5.2%	1.4%	1.3%	3.0%	5.6%	1.3%	1.3%	1.1%	2.7%	0.8%	1.1%	58.1%	
Samples	24865	26850	7595	5231	7534	15993	3516	3591	8304	14812	3871	3744	3590	7564	2605	3215	27732	32425	8737	6070	7934	15459	4329	3985	8913	16715	3824	4044	3371	7994	2400	3188	300000	
Output %	8.8%	8.3%	2.5%	2.0%	2.6%	5.2%	1.1%	1.4%	2.9%	4.8%	1.2%	1.4%	1.4%	2.1%	0.8%	0.9%	9.9%	10.3%	2.8%	2.1%	2.5%	5.6%	1.3%	1.3%	2.8%	5.6%	1.2%	1.4%	1.5%	2.8%	0.6%	0.8%	58.4%	
Samples	5293	4957	1490	1200	1546	3091	678	860	1735	2885	723	834	854	1287	497	559	5933	6190	1676	1252	1515	3343	796	792	1667	3345	723	852	873	1681	384	489	60000	
Output %	8.2%	9.3%	3.0%	1.8%	2.1%	5.7%	1.3%	1.1%	2.8%	5.3%	1.2%	1.1%	%6.0	2.7%	0.8%	1.2%	9.3%	10.4%	2.9%	2.0%	2.8%	5.0%	1.6%	1.2%	3.1%	5.3%	1.1%	1.3%	1.0%	2.7%	0.8%	0.9%	58.0%	
Samples	4901	5562	1829	1050	1284	3431	789	632	1690	3161	720	675	568	1626	503	729	5586	6267	1750	1192	1659	2977	986	735	1872	3208	657	756	603	1606	481	515	60000	
Output %	8.1%	9.1%	2.3%	1.4%	2.8%	5.0%	1.2%	1.2%	2.9%	4.9%	1.3%	1.2%	%6.0	2.4%	0.7%	1.2%	8.8%	11.9%	2.8%	2.2%	2.5%	5.1%	1.4%	1.4%	3.0%	5.7%	1.5%	1.6%	%6.0	2.4%	%6.0	1.3%	57.8%	
Samples	4875	5466	1368	828	1651	2979	729	708	1758	2932	804	726	567	1427	416	748	5268	7118	1670	1297	1528	3054	833	868	1819	3413	922	930	527	1462	523	786	60000	
Output %	8.5%	8.8%	2.3%	1.8%	2.7%	5.2%	1.0%	1.2%	2.8%	4.7%	1.4%	1.2%	1.3%	2.7%	%6.0	1.1%	9.4%	10.6%	3.1%	1.9%	3.1%	5.3%	1.6%	1.1%	2.8%	5.4%	1.1%	1.2%	1.0%	2.8%	%6.0	1.2%	58.8%	
Samples	5109	5305	1391	1087	1620	3093	579	706	1687	2844	811	737	798	1619	517	674	5656	6337	1856	1145	1854	3183	937	699	1661	3223	646	719	585	1703	533	716	60000	
Output %	7.8%	9.3%	2.5%	1.8%	2.4%	5.7%	1.2%	1.1%	2.4%	5.0%	1.4%	1.3%	1.3%	2.7%	1.1%	0.8%	8.8%	10.9%	3.0%	2.0%	2.3%	4.8%	1.3%	1.5%	3.2%	5.9%	1.5%	1.3%	1.3%	2.6%	0.8%	1.1%	59.1%	
Samples	4687	5560	1517	1066	1433	3399	741	685	1434	2990	813	772	803	1605	672	505	5289	6513	1785	1184	1378	2902	777	921	1894	3526	876	787	783	1542	479	682	60000	
Expected %	10.1%	10.1%	2.5%	0.6%	0.6%	5.1%	0.6%	0.6%	0.6%	5.1%	0.6%	0.6%	0.6%	0.6%	0.6%	0.6%	20.3%	20.3%	2.5%	0.6%	0.6%	5.1%	0.6%	0.6%	0.6%	5.1%	0.6%	0.6%	0.6%	0.6%	0.6%	0.6%	ted error =	
State	0	1	2	e	4	2	9	7	80	6	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	Accumula	
Ranked Interpret.	(16), (17)		(0), (1)		(5), (9), (21), (25)		(2), (18)		All other states 22	(3), (4), (6)-(8)	(10)-(15), (19), (20)	(22)-(24), (26)-(31)																						

*Figure 32.* Results for the MLP in the 5-atom micro-world, tested against the neutral premise. States are given in decimal format.

inked Interpret.	State	Expected %	Activations	Output %	Activations	Output %	Activations	Output %	Activations	Output %	Activations	Output %	Activations	Output %	_
6), (17)	0	12.5%	0.0007	1.2%	0.0005	0.8%	0.0008	1.4%	0.0007	1.2%	0.0012	1.9%	0.0039	1.3%	
	1	12.5%	0.000	%0.0	0.0000	0.0%	0.0000	0.0%	0.0001	0.2%	0.0000	0.0%	0.0001	0.0%	
), (1)	2	4.2%	0.0135	23.2%	0.0135	21.4%	0.0092	15.9%	0.0108	18.8%	0.0167	27.1%	0.0637	21.3%	
	e	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
), (9), (21), (25)	4	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
	5	8.3%	0.0055	9.5%	0.0065	10.3%	0.0063	10.9%	0.0056	9.7%	0.0043	7.0%	0.0282	9.5%	
), (18)	9	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
	7	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
other states 22	8	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
), (4), (6)-(8)	6	8.3%	0.0034	5.9%	0.0067	10.6%	0.0063	10.9%	0.0059	10.3%	0.0051	8.3%	0.0274	9.2%	
0)-(15), (19), (20)	10	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
2)-(24), (26)-(31)	11	0.0%	0.0008	1.4%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0046	1.5%	
	12	0.0%	0.0008	1.4%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0046	1.5%	
	13	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
	14	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
	15	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
	16	16.7%	0.0002	0.3%	0.0005	0.8%	0.0003	0.5%	0.0003	0.5%	0.0003	0.5%	0.0016	0.5%	
	17	16.7%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	18	4.2%	0.0070	12.0%	0.0077	12.2%	0.0092	15.9%	0.0070	12.2%	0.0087	14.1%	0.0396	13.3%	
	19	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0011	1.9%	0.0009	1.5%	0.0048	1.6%	
	20	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
	21	8.3%	0.0039	6.7%	0.0027	4.3%	0.0040	6.9%	0.0027	4.7%	0.0028	4.5%	0.0161	5.4%	
	22	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
	23	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
	24	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
	25	8.3%	0.0043	7.4%	0.0031	4.9%	0.0021	3.6%	0.0022	3.8%	0.0027	4.4%	0.0144	4.8%	
	26	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
	27	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
	28	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
	29	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
	30	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0011	1.9%	0.0009	1.5%	0.0048	1.6%	
	31	0.0%	0.0009	1.5%	0.0010	1.6%	0.0009	1.6%	0.0010	1.7%	0.0009	1.5%	0.0047	1.6%	
	Accumu	ated error =	0.0581	137.0%	0.0632	137.0%	0.0580	139.0%	0.0575	138.5%	0.0616	139.0%	0.2984	136.6%	+ 1.0

*Figure 33.* Results for the MLP in the 5-atom micro-world, tested against the neutral premise. States are given in decimal format.

anked Interpret.	State	Expected %	Activations	Output %	Activations	Output %	Activations	Output %	Activations	Output %	Activations	Output %	Activations	Output %	
16), (17)	0	10.6%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	-
	1	10.6%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
(1), (1)	2	3.5%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	m	0.7%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
5), (9), (21), (25)	4	0.7%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	S	7.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
1), (18)	9	0.7%	0.0000	0.0%	0.0000	0.0%	0.0001	14.3%	0.0000	0.0%	0.0000	0.0%	0.0001	1.3%	
	7	0.7%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
I other states 22	80	0.7%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
1), (4), (6)-(8)	6	7.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
(0)-(15), (19), (20)	10	0.7%	0.0000	0.0%	0.0001	16.7%	0.0000	0.0%	0.0002	100.0%	0.0013	21.3%	0.0016	20.0%	
12)-(24), (26)-(31)	11	0.7%	0.0000	0.0%	0.0002	33.3%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0002	2.5%	
	12	0.7%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	%0.0	0.0000	0.0%	
	13	0.7%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	14	0.7%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	15	0.7%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	16	14.1%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	17	14.1%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	18	3.5%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	19	0.7%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	20	0.7%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	21	7.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	%0.0	0.0000	0.0%	0.0000	0.0%	
	22	0.7%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	23	0.7%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	24	0.7%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	25	7.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	26	0.7%	0.0000	0.0%	0.0002	33.3%	0.0000	0.0%	0.0000	%0.0	0.0002	3.3%	0.0004	5.0%	
	27	0.7%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	28	0.7%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	
	29	0.7%	0.0000	0.0%	0.0000	0.0%	0.0006	85.7%	0.0000	0.0%	0.0000	0.0%	0.0006	7.5%	
	30	0.7%	0.0003	75.0%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0046	75.4%	0.0049	61.3%	
	31	0.7%	0.0001	25.0%	0.0001	16.7%	0.0000	0.0%	0.0000	0.0%	0.0000	0.0%	0.0002	2.5%	
	Accumu	ated error =	0.0004	197.0%	0.0006	194.2%	0.0007	197.0%	0.0002	198.4%	0.0061	195.6%	0.0080	190.0%	± 1.6
															1

*Figure 34.* Results for the MLP in the 5-atom micro-world, tested against the neutral premise. States are given in decimal format.



*Figure 35.* Results for the MLP in the 5-atom micro-world, tested against specific premises. States are given in decimal format.



*Figure 36.* Results for the MLP in the 5-atom micro-world, tested against specific premises. States are given in decimal format.



*Figure 37.* Results for the MLP in the 5-atom micro-world, tested against specific premises. States are given in decimal format.



**Appendix D: Blotzmann Machine Implementation** 

*Figure 38.* Simplified UML diagram for the object-oriented design of the Boltzmann machine.



*Figure 39.* Modified learning algorithm for the Boltzmann macine. The network is separated into layers and the annealing schedule is varied between phases.